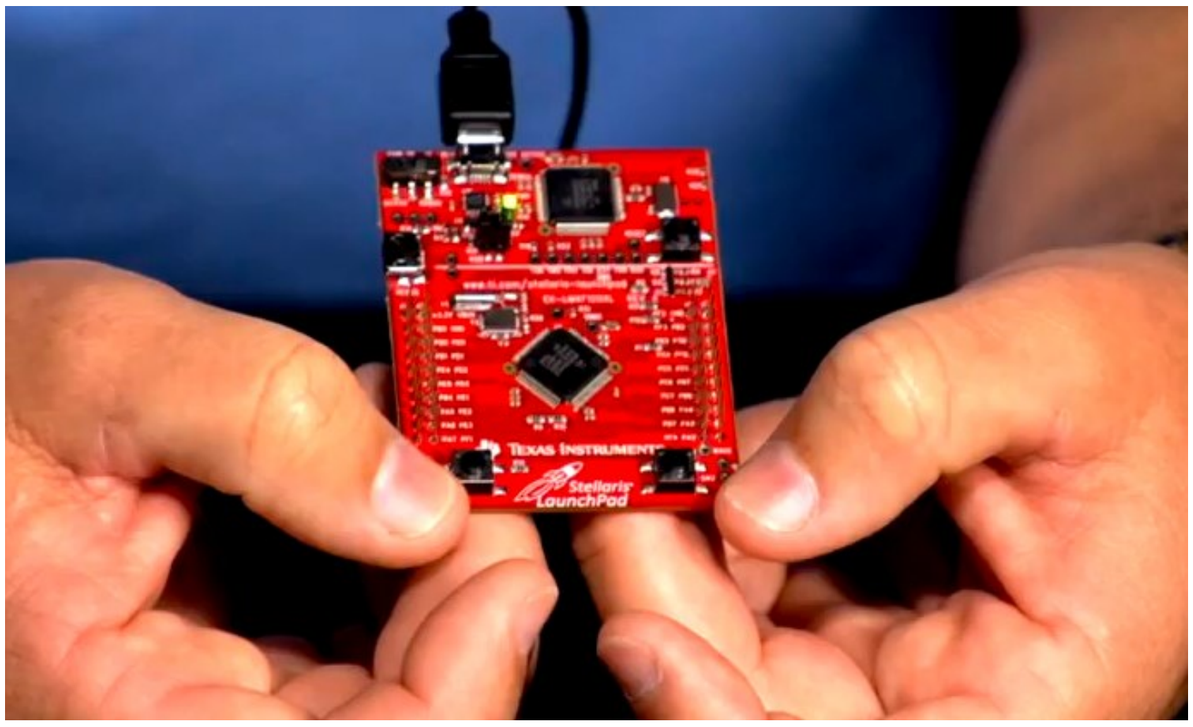
CPU-specific optimization

Example of a target CPU core:
ARM Cortex-M4F core inside
LM4F120H5QR microcontroller
in Stellaris LM4F120 Launchpad.



Example of a function
that we want to optimize:
adding 1000 integers mod 2^{32}.

Reference implementation:

```
int sum(int *x)
{
  int result = 0;
  int i;
  for (i = 0;i < 1000;++i)
    result += x[i];
  return result;
}
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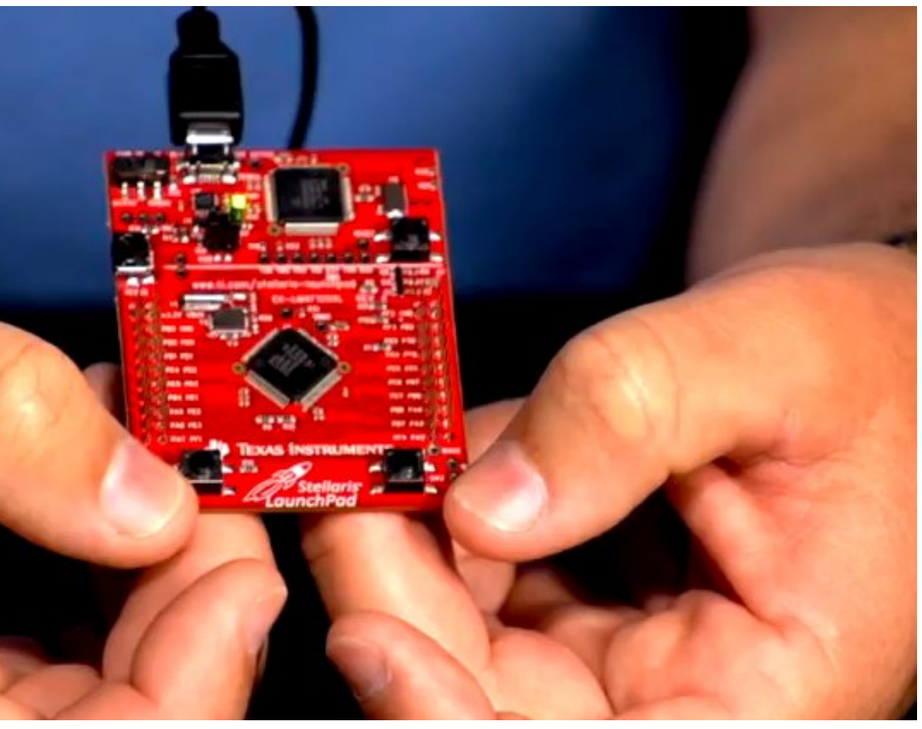

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Counting

```
static
  *cons
  = (vo
...

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int res
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  resul
```

Output
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core inside
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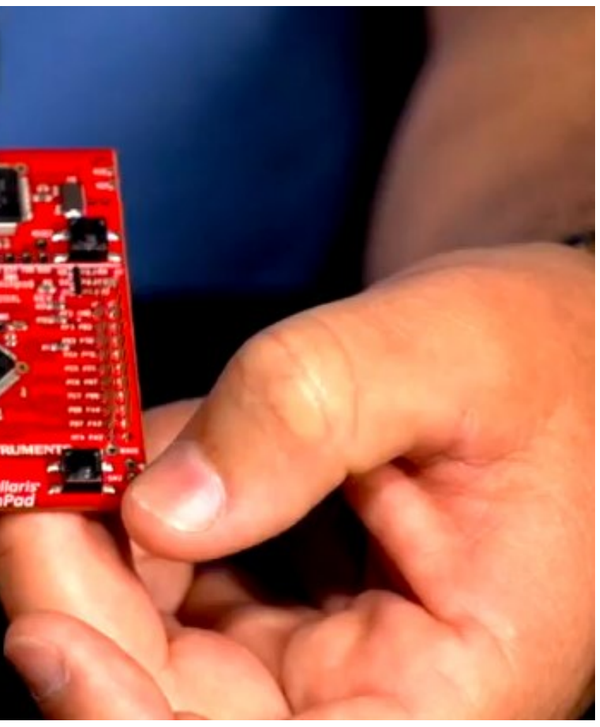
20 Launchpad.



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```


Counting cycles:

```
static volatile
  *const DWT_CYC
  = (void *) 0xE
...

int beforesum =
int result = sum
int aftersum = *
UARTprintf("sum
  result,aftersu
```

Output shows 801

Change 1000 to 5

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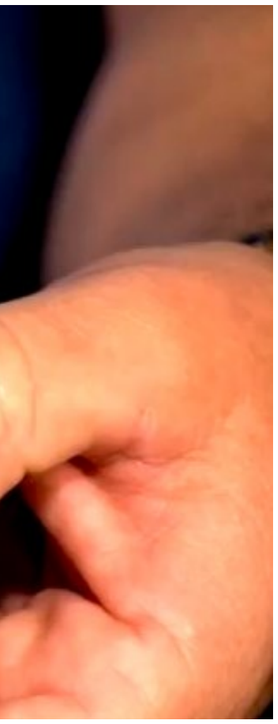

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Counting cycles:

```
static volatile unsigned
  *const DWT_CYCCNT
  = (void *) 0xE0001004;
...

int beforesum = *DWT_CYCC
int result = sum(x);
int aftersum = *DWT_CYCCN
UARTprintf("sum %d %d\n",
  result,aftersum-befores
```

Output shows 8012 cycles.
Change 1000 to 500: 4012.

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```
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"Okay, 8 cycles per addition
Um, are microcontrollers
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Bad approach:
Apply random "optimizations"
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until you get bored/frustrated.
Keep the fastest results.

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Try `-O1`: 8012 cycles.

Try `-O2`: 8012 cycles.

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  = (void *) 0xE0001004;
...

int beforesum = *DWT_CYCCNT;
int result = sum(x);
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UARTprintf("sum %d %d\n",
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Try `-O2`: 8012 cycles.

Try `-O3`: 8012 cycles.

g cycles:

```
volatile unsigned int
t DWT_CYCCNT
id *) 0xE0001004;


oresum = *DWT_CYCCNT;
ult = sum(x);
ersum = *DWT_CYCCNT;
ntf("sum %d %d\n",
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Try mov

```
int sum
{
  int re
  int i
  for (
    resu
  return
}
```

```
unsigned int
CNT
0001004;


*DWT_CYCCNT;
(x);
DWT_CYCCNT;
%d %d\n",
m-beforesum);
```

2 cycles.

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Try moving the po

```
int sum(int *x)
{
  int result = 0
  int i;
  for (i = 0;i <
    result += *x
  return result;
}
```

```
int

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Try moving the pointer:

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int sum(int *x)
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8010 cycles.

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5

Try counting down

```
int sum(int *x)
{
  int result = 0
  int i;
  for (i = 1000;
    result += *x
  return result;
}
```

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  return result;
}
```

8010 cycles.

Try counting down:

```
int sum(int *x)
{
  int result = 0;
  int i;
  for (i = 1000;i > 0;--i
    result += *x++;
  return result;
}
```

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(int *x)


esult = 0;

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Try usin

```
int sum
{
  int re
  int *
  while
    resu
  retur
}
```

pinter:

```
;

 1000;++i)
++;
```


Try counting down:

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  int i;
  for (i = 1000;i > 0;--i)
    result += *x++;
  return result;
}
```

8010 cycles.

Try using an end p

```
int sum(int *x)
{
  int result = 0
  int *y = x + 1
  while (x != y)
    result += *x
  return result;
}
```

Try counting down:

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  int result = 0;
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  for (i = 1000;i > 0;--i)
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Back to

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int sum
{
  int re
  int i
  for (
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    resu
  }
  return
}
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```
;


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++;
```


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  int *y = x + 1000;
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    result += *x++;
  return result;
}
```

8010 cycles.

Back to original. T

```
int sum(int *x)
{
  int result = 0
  int i;
  for (i = 0;i <
    result += x[
    result += x[
  }
  return result;
}
```

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int sum(int *x)
{
  int result = 0;
  int *y = x + 1000;
  while (x != y)
    result += *x++;
  return result;
}
```

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Back to original. Try unrolli

```
int sum(int *x)
{
  int result = 0;
  int i;
  for (i = 0;i < 1000;i +
    result += x[i];
    result += x[i + 1];
  }
  return result;
}
```

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  int result = 0;
  int *y = x + 1000;
  while (x != y)
    result += *x++;
  return result;
}
```

8010 cycles.

Back to original. Try unrolling:

```
int sum(int *x)
{
  int result = 0;
  int i;
  for (i = 0;i < 1000;i += 2) {
    result += x[i];
    result += x[i + 1];
  }
  return result;
}
```

Try using an end pointer:

```
int sum(int *x)
{
  int result = 0;
  int *y = x + 1000;
  while (x != y)
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  return result;
}
```

8010 cycles.

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  int i;
  for (i = 0;i < 1000;i += 2) {
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    result += x[i + 1];
  }
  return result;
}
```

5016 cycles.

g an end pointer:

```
(int *x)


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y = x + 1000;
 (x != y)
ult += *x++;
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```
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{
  int result = 0;
  int i;
  for (i = 0;i < 1000;i += 2) {
    result += x[i];
    result += x[i + 1];
  }
  return result;
}
```

5016 cycles.

```
int sum
{
  int re
  int i
  for (
    resu
    resu
    resu
    resu
  }
  return
}
```

pointer:

```
;
000;

++;
```


Back to original. Try unrolling:

```
int sum(int *x)
{
  int result = 0;
  int i;
  for (i = 0;i < 1000;i += 2) {
    result += x[i];
    result += x[i + 1];
  }
  return result;
}
```

5016 cycles.

```
int sum(int *x)
{
  int result = 0
  int i;
  for (i = 0;i <
    result += x[
    result += x[
    result += x[
    result += x[
    result += x[
  }
  return result;
}
```

Back to original. Try unrolling:

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int sum(int *x)
{
  int result = 0;
  int i;
  for (i = 0;i < 1000;i += 2) {
    result += x[i];
    result += x[i + 1];
  }
  return result;
}
```

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```
int sum(int *x)
{
  int result = 0;
  int i;
  for (i = 0;i < 1000;i +
    result += x[i];
    result += x[i + 1];
    result += x[i + 2];
    result += x[i + 3];
    result += x[i + 4];
  }
  return result;
}
```

Back to original. Try unrolling:

```
int sum(int *x)
{
  int result = 0;
  int i;
  for (i = 0;i < 1000;i += 2) {
    result += x[i];
    result += x[i + 1];
  }
  return result;
}
```

5016 cycles.


```
int sum(int *x)
{
  int result = 0;
  int i;
  for (i = 0;i < 1000;i += 5) {
    result += x[i];
    result += x[i + 1];
    result += x[i + 2];
    result += x[i + 3];
    result += x[i + 4];
  }
  return result;
}
```

original. Try unrolling:

```
(int *x)


esult = 0;
;
i = 0;i < 1000;i += 2) {
ult += x[i];
ult += x[i + 1];


n result;
```

cles.

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```
int sum(int *x)
{
  int result = 0;
  int i;
  for (i = 0;i < 1000;i += 5) {
    result += x[i];
    result += x[i + 1];
    result += x[i + 2];
    result += x[i + 3];
    result += x[i + 4];
  }
  return result;
}
```

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4016 cyc

Try unrolling:

```
;


 1000;i += 2) {
i];
i + 1];
```



```
int sum(int *x)
{
  int result = 0;
  int i;
  for (i = 0;i < 1000;i += 5) {
    result += x[i];
    result += x[i + 1];
    result += x[i + 2];
    result += x[i + 3];
    result += x[i + 4];
  }
  return result;
}
```

4016 cycles. Are v

ng:

```
= 2) {
```

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```
int sum(int *x)
{
  int result = 0;
  int i;
  for (i = 0;i < 1000;i += 5) {
    result += x[i];
    result += x[i + 1];
    result += x[i + 2];
    result += x[i + 3];
    result += x[i + 4];
  }
  return result;
}
```

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4016 cycles. Are we done n

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4016 cycles. Are we done now?

Most random "optimizations"
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Can spend time trying more.
Does frustration level tell us
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}
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Good approach:
Figure out lower bound for
cycles spent on arithmetic etc.
Understand gap between
lower bound and observed time.

Let's try this approach.

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;
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Inputs and output of ADD a
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Each element of `x` array nee
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Basic load instruction: LDR
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Can achieve this speed
in other ways (LDRD, LDM)
but nothing seems faster.

Lower bound for n LDR + n
$2n+1$ cycles, including
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Why observed time is higher
non-consecutive LDRs;
costs of manipulating `i`.

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```
y = x +
p = x
result =

loop:

  xi9 =
  xi8 =
  xi7 =
  xi6 =
  xi5 =
  xi4 =
  xi3 =
  xi2 =
```

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2281 cycles using

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y = x + 4000
p = x
result = 0

loop:

  xi9 = *(uint32
  xi8 = *(uint32
  xi7 = *(uint32
  xi6 = *(uint32
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  xi4 = *(uint32
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2281 cycles using `ldr.w`:

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```
y = x + 4000
p = x
result = 0


loop:


  xi9 = *(uint32 *) (p + 76)
  xi8 = *(uint32 *) (p + 72)
  xi7 = *(uint32 *) (p + 68)
  xi6 = *(uint32 *) (p + 64)
  xi5 = *(uint32 *) (p + 60)
  xi4 = *(uint32 *) (p + 56)
  xi3 = *(uint32 *) (p + 52)
  xi2 = *(uint32 *) (p + 48)
```

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```

```
  xi1 =
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  xi9 =
  xi8 =
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```
  xi1 = *(uint32
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  result += xi9
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  result += xi6
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```



```
  xi1 = *(uint32 *) (p + 44)
  xi0 = *(uint32 *) (p + 40)
  result += xi9
  result += xi8
  result += xi7
  result += xi6
  result += xi5
  result += xi4
  result += xi3
  result += xi2
  result += xi1
  result += xi0
  xi9 = *(uint32 *) (p + 36)
  xi8 = *(uint32 *) (p + 32)
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```

cles using `ldr.w`:

```
 4000


= 0




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xi1 = *(uint32 *) (p + 44)
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result += xi9
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result += xi6
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xi9 = *(uint32 *) (p + 36)
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```

```
xi6 =
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```

```
ldr.w:

 *) (p + 76)
*) (p + 72)
*) (p + 68)
*) (p + 64)
*) (p + 60)
*) (p + 56)
*) (p + 52)
*) (p + 48)
```



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    xi1 = *(uint32 *) (p + 44)
    xi0 = *(uint32 *) (p + 40)
    result += xi9
    result += xi8
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    result += xi7
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    result += xi5
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```
76)
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```
    xi1 = *(uint32 *) (p + 44)
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    xi6 = *(uint32 *) (p +
    xi5 = *(uint32 *) (p +
    xi4 = *(uint32 *) (p +
    xi3 = *(uint32 *) (p +
    xi2 = *(uint32 *) (p +
    xi1 = *(uint32 *) (p +
    xi0 = *(uint32 *) p; p
    result += xi9
    result += xi8
    result += xi7
    result += xi6
    result += xi5
    result += xi4
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```
xi1 = *(uint32 *) (p + 44)
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xi9 = *(uint32 *) (p + 36)
xi8 = *(uint32 *) (p + 32)
xi7 = *(uint32 *) (p + 28)
```



```
xi6 = *(uint32 *) (p + 24)
xi5 = *(uint32 *) (p + 20)
xi4 = *(uint32 *) (p + 16)
xi3 = *(uint32 *) (p + 12)
xi2 = *(uint32 *) (p + 8)
xi1 = *(uint32 *) (p + 4)
xi0 = *(uint32 *) p; p += 160
result += xi9
result += xi8
result += xi7
result += xi6
result += xi5
result += xi4
result += xi3
result += xi2
```

```
 *(uint32 *) (p + 44)
 *(uint32 *) (p + 40)
t += xi9
t += xi8
t += xi7
t += xi6
t += xi5
t += xi4
t += xi3
t += xi2
t += xi1
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```

```
    result
    result
    xi9 =
    xi8 =
    xi7 =
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    xi5 =
    xi4 =
    xi3 =
    xi2 =
    xi1 =
    xi0 =
    resul
    result
    result
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```
*) (p + 44)
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  result += xi9
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  result += xi1
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  xi6 = *(uint32
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  xi4 = *(uint32
  xi3 = *(uint32
  xi2 = *(uint32
  xi1 = *(uint32
  xi0 = *(uint32
  result += xi9
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```

```
44)
40)
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32)
28)
```

15

```
    xi6 = *(uint32 *) (p + 24)
    xi5 = *(uint32 *) (p + 20)
    xi4 = *(uint32 *) (p + 16)
    xi3 = *(uint32 *) (p + 12)
    xi2 = *(uint32 *) (p + 8)
    xi1 = *(uint32 *) (p + 4)
    xi0 = *(uint32 *) p; p += 160
    result += xi9
    result += xi8
    result += xi7
    result += xi6
    result += xi5
    result += xi4
    result += xi3
    result += xi2
```

16

```
    result += xi1
    result += xi0
    xi9 = *(uint32 *) (p -
    xi8 = *(uint32 *) (p -
    xi7 = *(uint32 *) (p -
    xi6 = *(uint32 *) (p -
    xi5 = *(uint32 *) (p -
    xi4 = *(uint32 *) (p -
    xi3 = *(uint32 *) (p -
    xi2 = *(uint32 *) (p -
    xi1 = *(uint32 *) (p -
    xi0 = *(uint32 *) (p -
    result += xi9
    result += xi8
    result += xi7
```

```
xi6 = *(uint32 *) (p + 24)
xi5 = *(uint32 *) (p + 20)
xi4 = *(uint32 *) (p + 16)
xi3 = *(uint32 *) (p + 12)
xi2 = *(uint32 *) (p + 8)
xi1 = *(uint32 *) (p + 4)
xi0 = *(uint32 *) p; p += 160
result += xi9
result += xi8
result += xi7
result += xi6
result += xi5
result += xi4
result += xi3
result += xi2
```



```
result += xi1
result += xi0
xi9 = *(uint32 *) (p - 4)
xi8 = *(uint32 *) (p - 8)
xi7 = *(uint32 *) (p - 12)
xi6 = *(uint32 *) (p - 16)
xi5 = *(uint32 *) (p - 20)
xi4 = *(uint32 *) (p - 24)
xi3 = *(uint32 *) (p - 28)
xi2 = *(uint32 *) (p - 32)
xi1 = *(uint32 *) (p - 36)
xi0 = *(uint32 *) (p - 40)
result += xi9
result += xi8
result += xi7
```

16

```
 *(uint32 *) (p + 24)
 *(uint32 *) (p + 20)
 *(uint32 *) (p + 16)
 *(uint32 *) (p + 12)
 *(uint32 *) (p + 8)
 *(uint32 *) (p + 4)
 *(uint32 *) p; p += 160
t += xi9
t += xi8
t += xi7
t += xi6
t += xi5
t += xi4
t += xi3
t += xi2
```

17

```
   result += xi1
   result += xi0
   xi9 = *(uint32 *) (p - 4)
   xi8 = *(uint32 *) (p - 8)
   xi7 = *(uint32 *) (p - 12)
   xi6 = *(uint32 *) (p - 16)
   xi5 = *(uint32 *) (p - 20)
   xi4 = *(uint32 *) (p - 24)
   xi3 = *(uint32 *) (p - 28)
   xi2 = *(uint32 *) (p - 32)
   xi1 = *(uint32 *) (p - 36)
   xi0 = *(uint32 *) (p - 40)
   result += xi9
   result += xi8
   result += xi7
```

```
   resul
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   resul
   resul
   xi9 =
   xi8 =
   xi7 =
   xi6 =
   xi5 =
   xi4 =
   xi3 =
   xi2 =
```

```
*) (p + 24)
*) (p + 20)
*) (p + 16)
*) (p + 12)
*) (p + 8)
*) (p + 4)
*) p; p += 160
```



```
  result += xi1
  result += xi0
  xi9 = *(uint32 *) (p - 4)
  xi8 = *(uint32 *) (p - 8)
  xi7 = *(uint32 *) (p - 12)
  xi6 = *(uint32 *) (p - 16)
  xi5 = *(uint32 *) (p - 20)
  xi4 = *(uint32 *) (p - 24)
  xi3 = *(uint32 *) (p - 28)
  xi2 = *(uint32 *) (p - 32)
  xi1 = *(uint32 *) (p - 36)
  xi0 = *(uint32 *) (p - 40)
  result += xi9
  result += xi8
  result += xi7
```

```
  result += xi6
  result += xi5
  result += xi4
  result += xi3
  result += xi2
  result += xi1
  result += xi0
  xi9 = *(uint32
  xi8 = *(uint32
  xi7 = *(uint32
  xi6 = *(uint32
  xi5 = *(uint32
  xi4 = *(uint32
  xi3 = *(uint32
  xi2 = *(uint32
```

```
24)
20)
16)
12)
8)
4)
+= 160
```

```
   result += xi1
   result += xi0
   xi9 = *(uint32 *) (p - 4)
   xi8 = *(uint32 *) (p - 8)
   xi7 = *(uint32 *) (p - 12)
   xi6 = *(uint32 *) (p - 16)
   xi5 = *(uint32 *) (p - 20)
   xi4 = *(uint32 *) (p - 24)
   xi3 = *(uint32 *) (p - 28)
   xi2 = *(uint32 *) (p - 32)
   xi1 = *(uint32 *) (p - 36)
   xi0 = *(uint32 *) (p - 40)
   result += xi9
   result += xi8
   result += xi7
```

```
   result += xi6
   result += xi5
   result += xi4
   result += xi3
   result += xi2
   result += xi1
   result += xi0
   xi9 = *(uint32 *) (p -
   xi8 = *(uint32 *) (p -
   xi7 = *(uint32 *) (p -
   xi6 = *(uint32 *) (p -
   xi5 = *(uint32 *) (p -
   xi4 = *(uint32 *) (p -
   xi3 = *(uint32 *) (p -
   xi2 = *(uint32 *) (p -
```

```
result += xi1
result += xi0
xi9 = *(uint32 *) (p - 4)
xi8 = *(uint32 *) (p - 8)
xi7 = *(uint32 *) (p - 12)
xi6 = *(uint32 *) (p - 16)
xi5 = *(uint32 *) (p - 20)
xi4 = *(uint32 *) (p - 24)
xi3 = *(uint32 *) (p - 28)
xi2 = *(uint32 *) (p - 32)
xi1 = *(uint32 *) (p - 36)
xi0 = *(uint32 *) (p - 40)
result += xi9
result += xi8
result += xi7
```



```
result += xi6
result += xi5
result += xi4
result += xi3
result += xi2
result += xi1
result += xi0
xi9 = *(uint32 *) (p - 44)
xi8 = *(uint32 *) (p - 48)
xi7 = *(uint32 *) (p - 52)
xi6 = *(uint32 *) (p - 56)
xi5 = *(uint32 *) (p - 60)
xi4 = *(uint32 *) (p - 64)
xi3 = *(uint32 *) (p - 68)
xi2 = *(uint32 *) (p - 72)
```

17

```
t += xi1
t += xi0
 *(uint32 *) (p - 4)
 *(uint32 *) (p - 8)
 *(uint32 *) (p - 12)
 *(uint32 *) (p - 16)
 *(uint32 *) (p - 20)
 *(uint32 *) (p - 24)
 *(uint32 *) (p - 28)
 *(uint32 *) (p - 32)
 *(uint32 *) (p - 36)
 *(uint32 *) (p - 40)
t += xi9
t += xi8
t += xi7
```

18

```
   result += xi6
   result += xi5
   result += xi4
   result += xi3
   result += xi2
   result += xi1
   result += xi0
   xi9 = *(uint32 *) (p - 44)
   xi8 = *(uint32 *) (p - 48)
   xi7 = *(uint32 *) (p - 52)
   xi6 = *(uint32 *) (p - 56)
   xi5 = *(uint32 *) (p - 60)
   xi4 = *(uint32 *) (p - 64)
   xi3 = *(uint32 *) (p - 68)
   xi2 = *(uint32 *) (p - 72)
```

```
   xi1 =
   xi0 =
   result
   result
   result
   result
   result
   result
   result
   result
   result
   result
 goto loo
```

17

```
*) (p - 4)
*) (p - 8)
*) (p - 12)
*) (p - 16)
*) (p - 20)
*) (p - 24)
*) (p - 28)
*) (p - 32)
*) (p - 36)
*) (p - 40)
```

```
result += xi6
result += xi5
result += xi4
result += xi3
result += xi2
result += xi1
result += xi0
xi9 = *(uint32 *) (p - 44)
xi8 = *(uint32 *) (p - 48)
xi7 = *(uint32 *) (p - 52)
xi6 = *(uint32 *) (p - 56)
xi5 = *(uint32 *) (p - 60)
xi4 = *(uint32 *) (p - 64)
xi3 = *(uint32 *) (p - 68)
xi2 = *(uint32 *) (p - 72)
```

18

```
xi1 = *(uint32
xi0 = *(uint32
result += xi9
result += xi8
result += xi7
result += xi6
result += xi5
result += xi4
result += xi3
result += xi2
result += xi1
result += xi0
                =?
goto loop if !=
```

```
4)
8)
12)
16)
20)
24)
28)
32)
36)
40)
```



```
result += xi6
result += xi5
result += xi4
result += xi3
result += xi2
result += xi1
result += xi0
xi9 = *(uint32 *) (p - 44)
xi8 = *(uint32 *) (p - 48)
xi7 = *(uint32 *) (p - 52)
xi6 = *(uint32 *) (p - 56)
xi5 = *(uint32 *) (p - 60)
xi4 = *(uint32 *) (p - 64)
xi3 = *(uint32 *) (p - 68)
xi2 = *(uint32 *) (p - 72)
```



```
  xi1 = *(uint32 *) (p -
  xi0 = *(uint32 *) (p -
  result += xi9
  result += xi8
  result += xi7
  result += xi6
  result += xi5
  result += xi4
  result += xi3
  result += xi2
  result += xi1
  result += xi0

                 =? p - y
goto loop if !=
```

```
result += xi6
result += xi5
result += xi4
result += xi3
result += xi2
result += xi1
result += xi0
xi9 = *(uint32 *) (p - 44)
xi8 = *(uint32 *) (p - 48)
xi7 = *(uint32 *) (p - 52)
xi6 = *(uint32 *) (p - 56)
xi5 = *(uint32 *) (p - 60)
xi4 = *(uint32 *) (p - 64)
xi3 = *(uint32 *) (p - 68)
xi2 = *(uint32 *) (p - 72)
```



```
xi1 = *(uint32 *) (p - 76)
xi0 = *(uint32 *) (p - 80)
result += xi9
result += xi8
result += xi7
result += xi6
result += xi5
result += xi4
result += xi3
result += xi2
result += xi1
result += xi0

                =? p - y
goto loop if !=
```

```
t += xi6
t += xi5
t += xi4
t += xi3
t += xi2
t += xi1
t += xi0
 *(uint32 *) (p - 44)
*(uint32 *) (p - 48)
*(uint32 *) (p - 52)
*(uint32 *) (p - 56)
*(uint32 *) (p - 60)
*(uint32 *) (p - 64)
*(uint32 *) (p - 68)
*(uint32 *) (p - 72)
```



```
  xi1 = *(uint32 *) (p - 76)
  xi0 = *(uint32 *) (p - 80)
  result += xi9
  result += xi8
  result += xi7
  result += xi6
  result += xi5
  result += xi4
  result += xi3
  result += xi2
  result += xi1
  result += xi0

                 =? p - y
goto loop if !=
```

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```
*) (p - 44)
*) (p - 48)
*) (p - 52)
*) (p - 56)
*) (p - 60)
*) (p - 64)
*) (p - 68)
*) (p - 72)
```



```
xi1 = *(uint32 *) (p - 76)
xi0 = *(uint32 *) (p - 80)
result += xi9
result += xi8
result += xi7
result += xi6
result += xi5
result += xi4
result += xi3
result += xi2
result += xi1
result += xi0

                =? p - y
goto loop if !=
```

Wikipedia: “By th
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```
18                                                                  19
        xi1 = *(uint32 *) (p - 76)
        xi0 = *(uint32 *) (p - 80)
        result += xi9
        result += xi8
        result += xi7
        result += xi6
        result += xi5
44)     result += xi4
48)       result += xi3
52)       result += xi2
56)       result += xi1
60)       result += xi0
64)
68)                  =? p - y
72)   goto loop if !=
```

Wikipedia: “By the late 199
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```
xi1 = *(uint32 *) (p - 76)
xi0 = *(uint32 *) (p - 80)
result += xi9
result += xi8
result += xi7
result += xi6
result += xi5
result += xi4
result += xi3
result += xi2
result += xi1
result += xi0

            =? p - y
goto loop if !=
```


Wikipedia: “By the late 1990s for even performance sensitive code, optimizing compilers exceeded the performance of human experts.”

```
xi1 = *(uint32 *) (p - 76)
xi0 = *(uint32 *) (p - 80)
result += xi9
result += xi8
result += xi7
result += xi6
result += xi5
result += xi4
  result += xi3
  result += xi2
  result += xi1
  result += xi0

                =? p - y
goto loop if !=
```


Wikipedia: "By the late 1990s for even performance sensitive code, optimizing compilers exceeded the performance of human experts."

Reality: The fastest software today relies on human experts understanding the CPU.

Cannot trust compiler to optimize instruction selection.

Cannot trust compiler to optimize instruction scheduling.

Cannot trust compiler to optimize register allocation.

```
 *(uint32 *) (p - 76)
 *(uint32 *) (p - 80)
t += xi9
t += xi8
t += xi7
t += xi6
t += xi5
t += xi4
t += xi3
t += xi2
t += xi1
t += xi0

       =? p - y
op if !=
```


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```
*) (p - 76)
*) (p - 80)
```

```
p - y
```

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The big picture

CPUs are evolving farther and farther from naive models

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The big picture

CPUs are evolving
farther and farther away
from naive models of CPUs.

Minor optimization challenges:

- Pipelining.
- Superscalar processing.

Major optimization challenges:

- Vectorization.
- Many threads; many cores.
- The memory hierarchy;
  the ring; the mesh.
- Larger-scale parallelism.
- Larger-scale networking.

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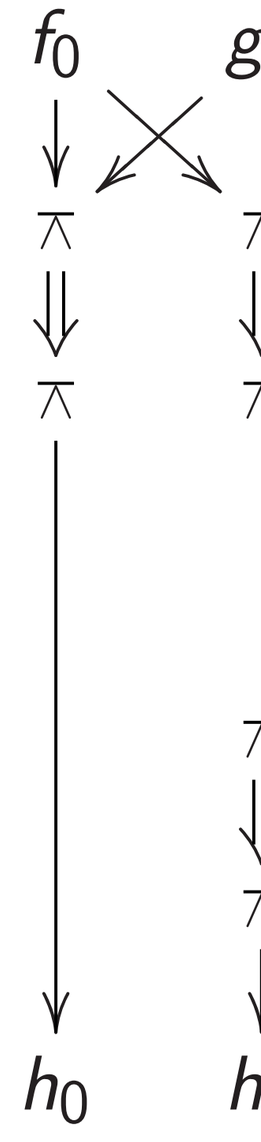

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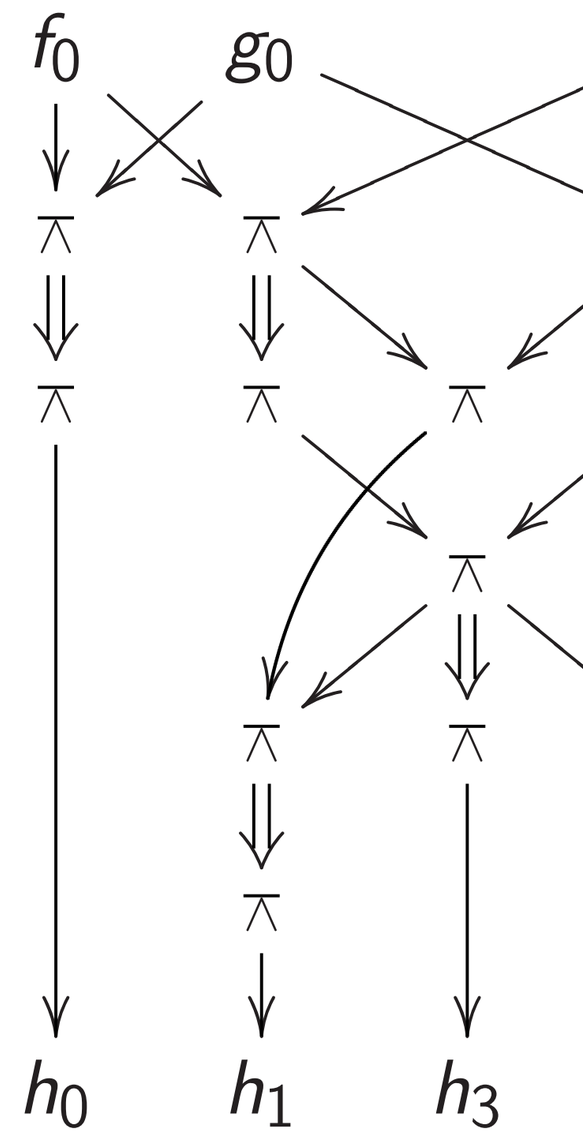
CPU design in a n



Gates $\barwedge : a, b \mapsto 1$
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Major optimization challenges:

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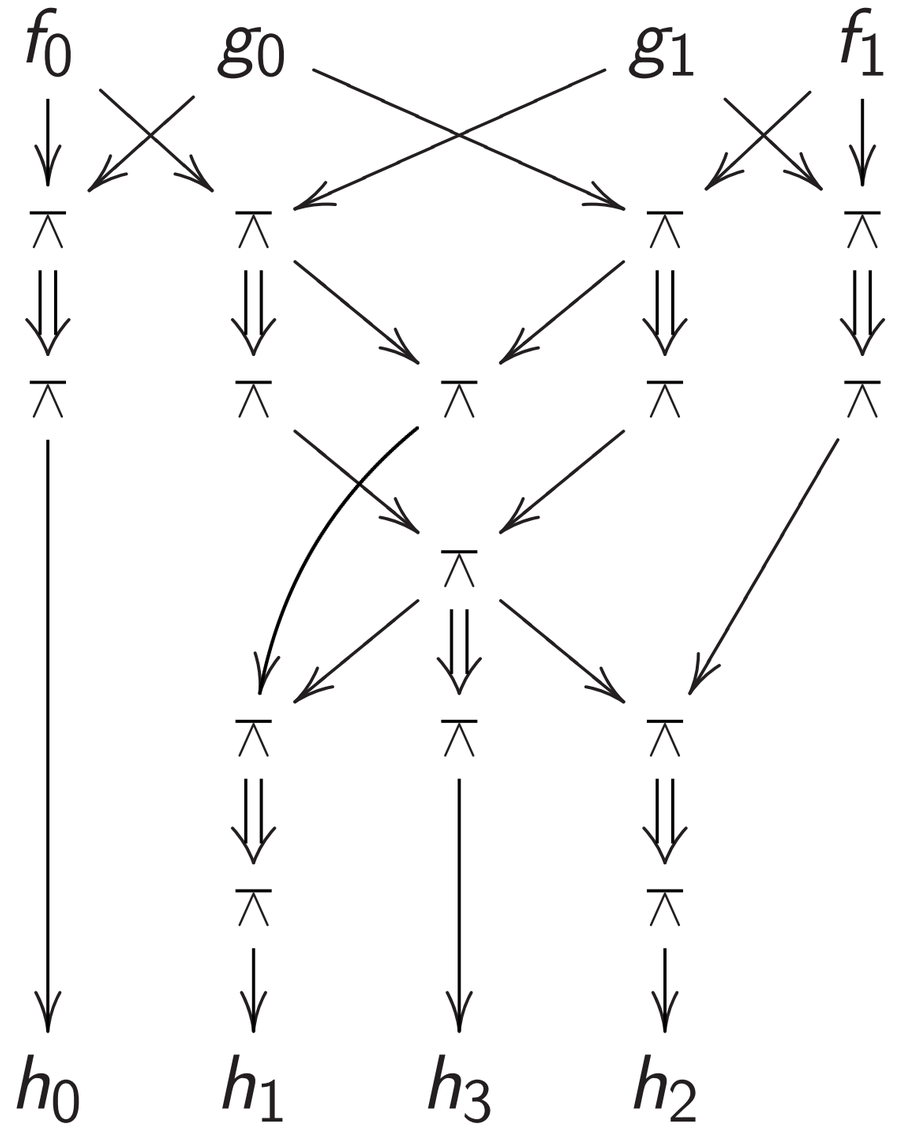
- Larger-scale parallelism.
- Larger-scale networking.

CPU design in a nutshell



Gates $\curlywedge : a, b \mapsto 1 - ab$ com
product $h_0 + 2h_1 + 4h_2 + 8$
of integers $f_0 + 2f_1, g_0 + 2g$

The big picture

CPUs are evolving
farther and farther away
from naive models of CPUs.

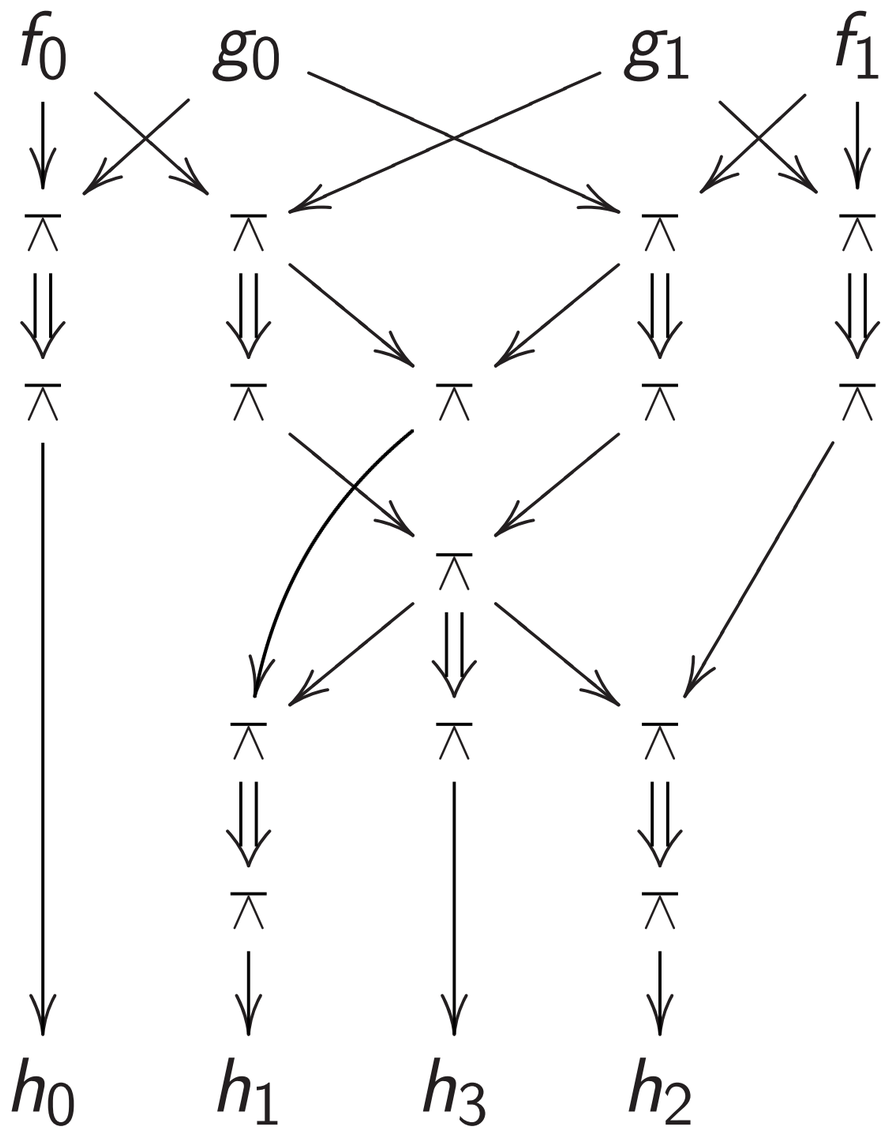
Minor optimization challenges:

- Pipelining.
- Superscalar processing.

Major optimization challenges:

- Vectorization.
- Many threads; many cores.
- The memory hierarchy;
  the ring; the mesh.
- Larger-scale parallelism.
- Larger-scale networking.

CPU design in a nutshell



Gates $\barwedge : a, b \mapsto 1 - ab$ computing
product $h_0 + 2h_1 + 4h_2 + 8h_3$
of integers $f_0 + 2f_1, g_0 + 2g_1$.

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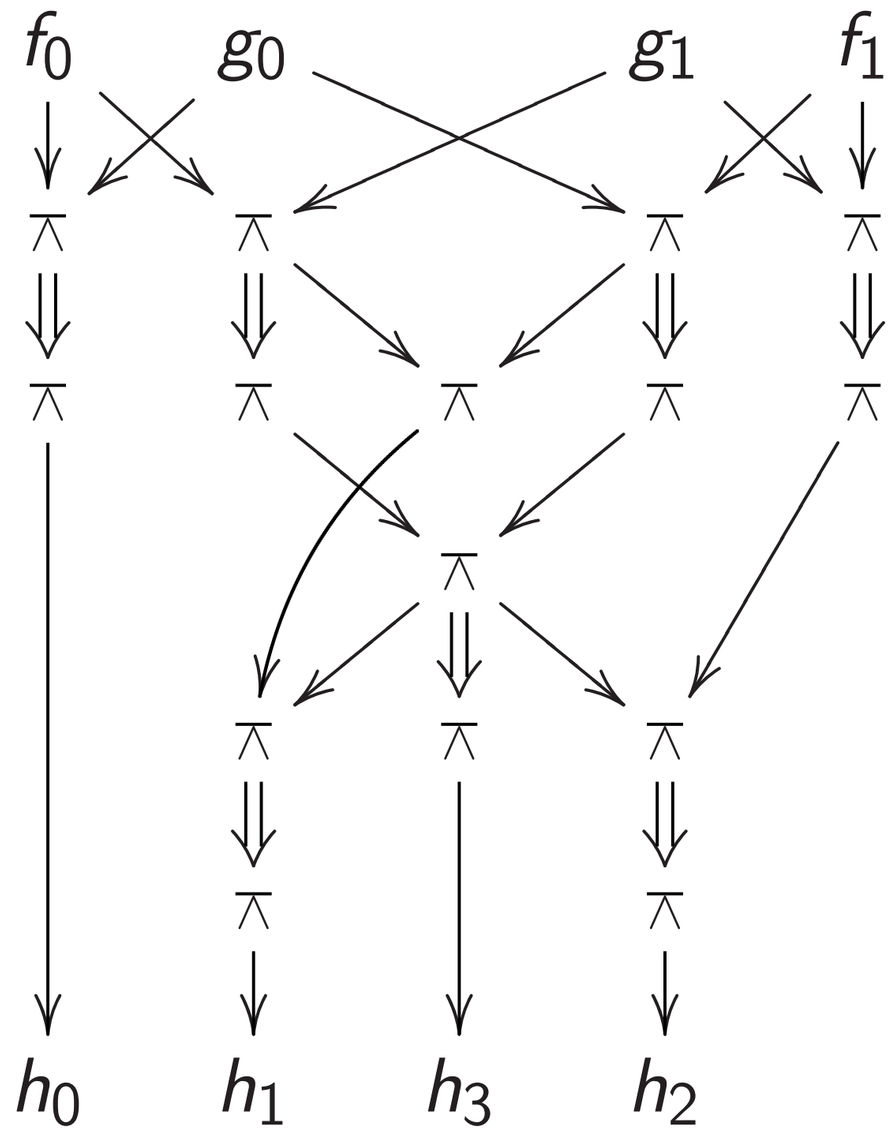
CPU design in a nutshell



Gates $\barwedge : a, b \mapsto 1 - ab$ computing product $h_0 + 2h_1 + 4h_2 + 8h_3$ of integers $f_0 + 2f_1, g_0 + 2g_1$.

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If $f_0, f_1,$ &

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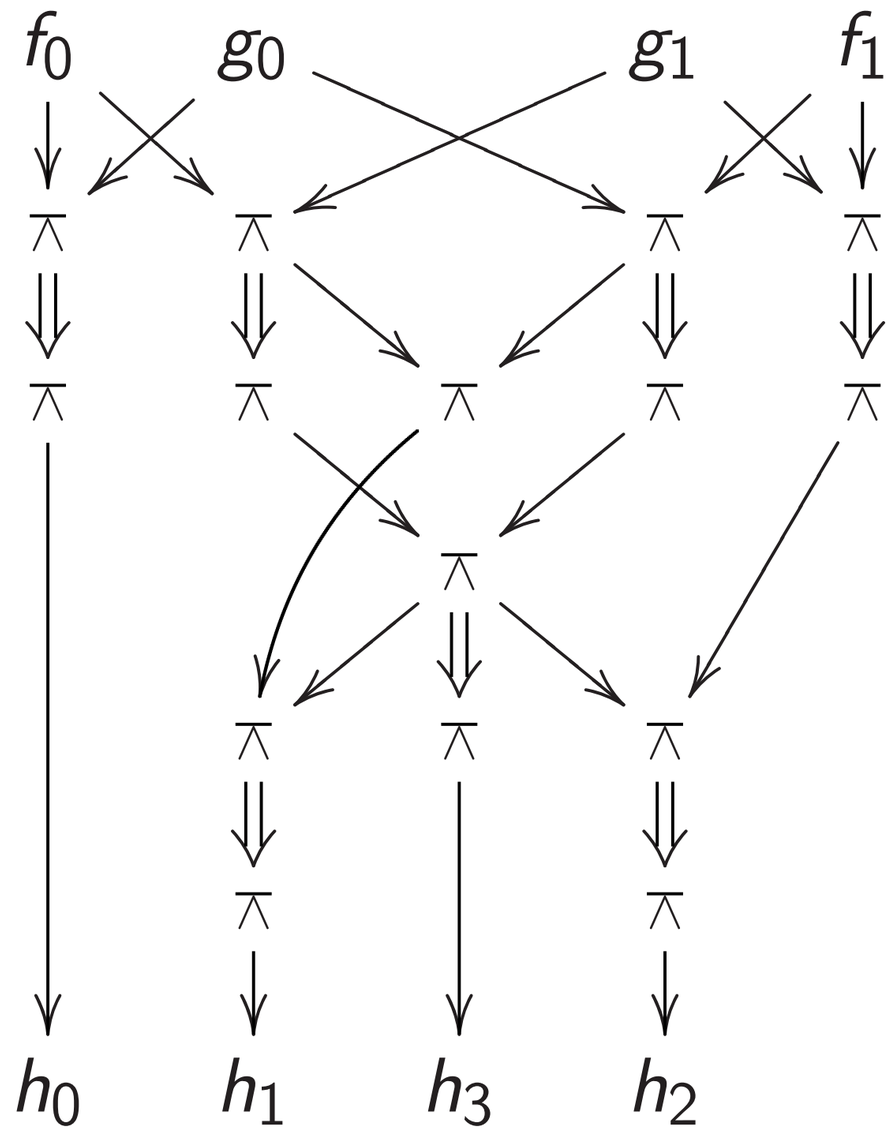
CPU design in a nutshell



Gates $\barwedge : a, b \mapsto 1 - ab$ computing product $h_0 + 2h_1 + 4h_2 + 8h_3$ of integers $f_0 + 2f_1, g_0 + 2g_1$.

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If f_0, f_1, g_0, g_1 are

then h_0, h_1, h_2, h_3

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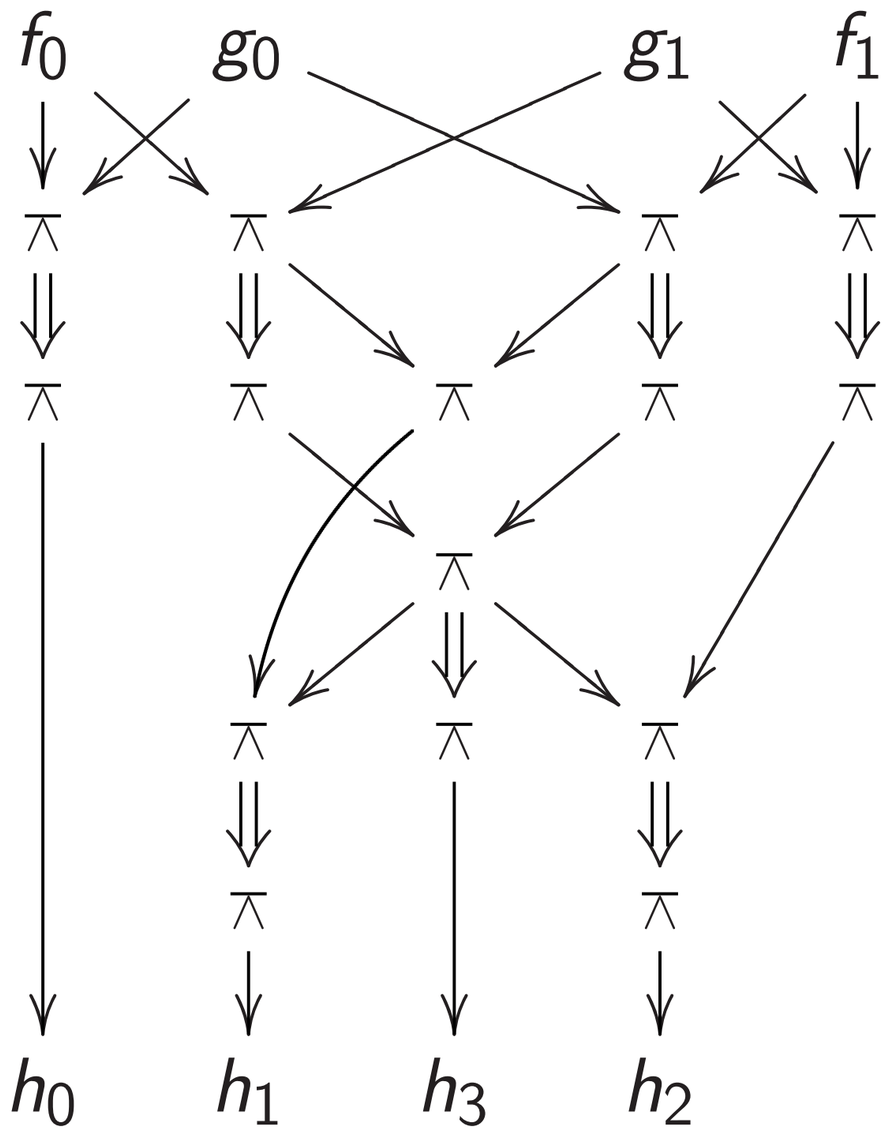
CPU design in a nutshell

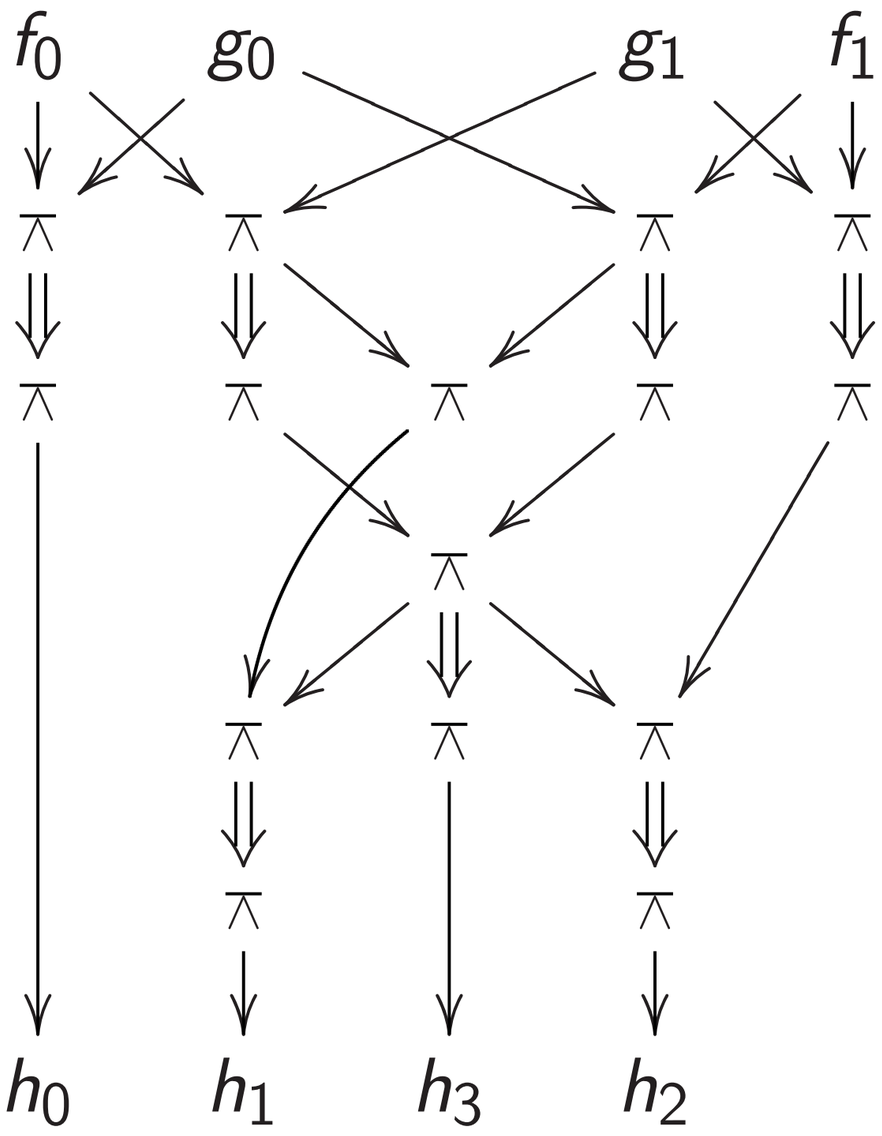


Gates $\barwedge : a, b \mapsto 1 - ab$ computing product $h_0 + 2h_1 + 4h_2 + 8h_3$ of integers $f_0 + 2f_1, g_0 + 2g_1$.

Electricity takes time to percolate through wires and

If f_0, f_1, g_0, g_1 are stable then h_0, h_1, h_2, h_3 are stable a few moments later.

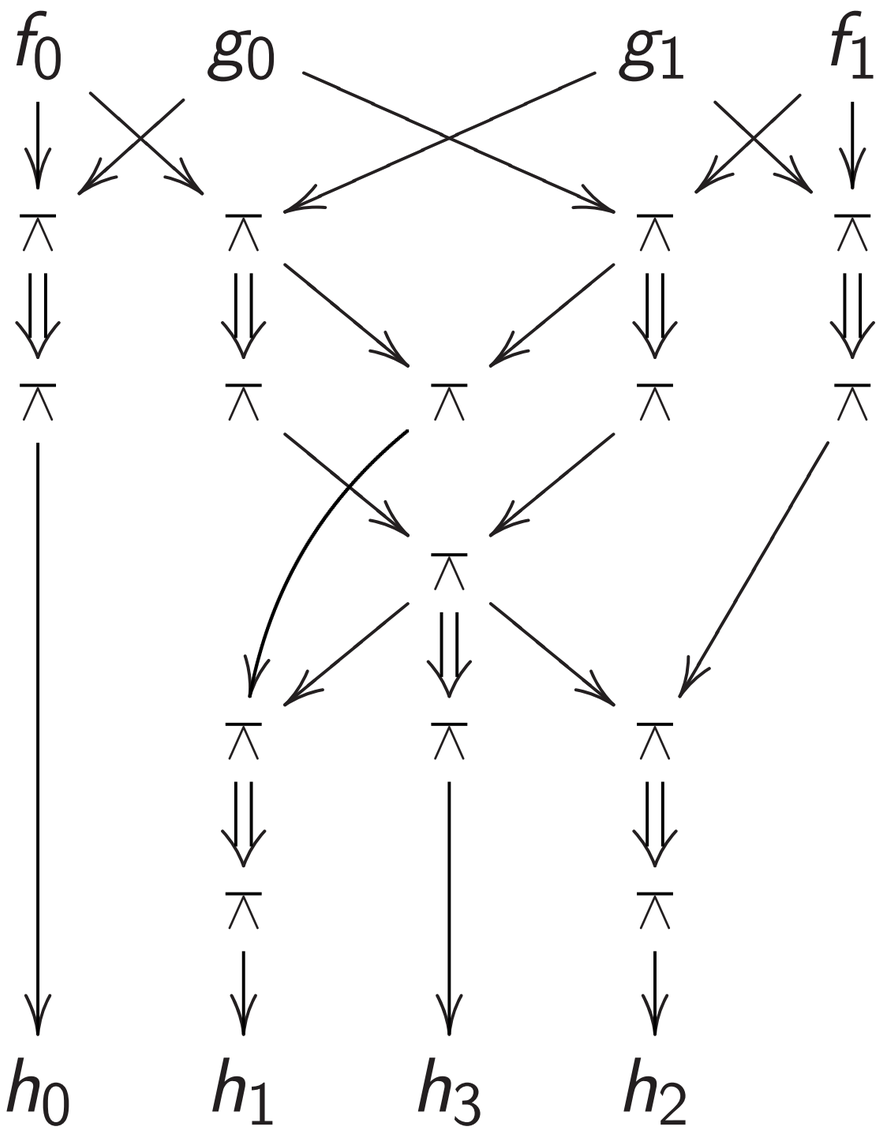
CPU design in a nutshell



Gates $\barwedge : a, b \mapsto 1 - ab$ computing
product $h_0 + 2h_1 + 4h_2 + 8h_3$
of integers $f_0 + 2f_1, g_0 + 2g_1$.

Electricity takes time to
percolate through wires and gates.
If f_0, f_1, g_0, g_1 are stable
then h_0, h_1, h_2, h_3 are stable
a few moments later.

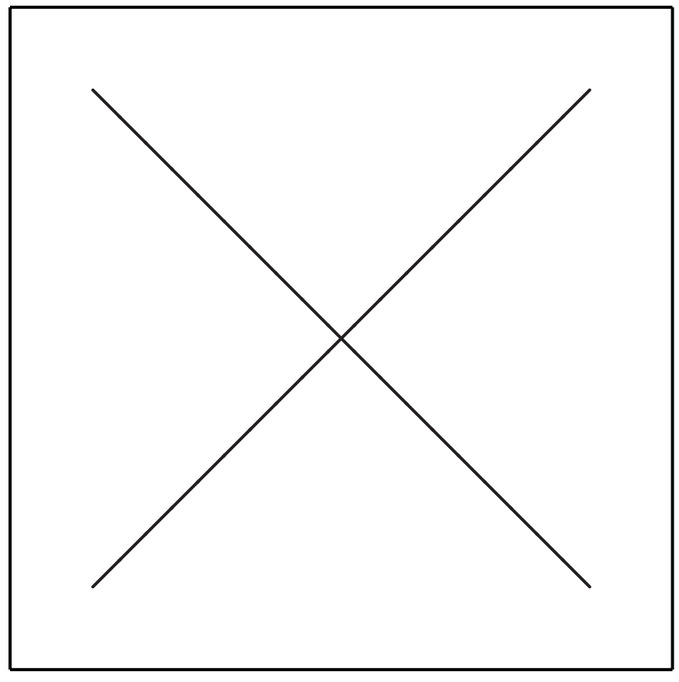
CPU design in a nutshell



Gates $\barwedge : a, b \mapsto 1 - ab$ computing
product $h_0 + 2h_1 + 4h_2 + 8h_3$
of integers $f_0 + 2f_1, g_0 + 2g_1$.

Electricity takes time to
percolate through wires and gates.
If f_0, f_1, g_0, g_1 are stable
then h_0, h_1, h_2, h_3 are stable
a few moments later.

Build circuit with more gates
to multiply (e.g.) 32-bit integers:



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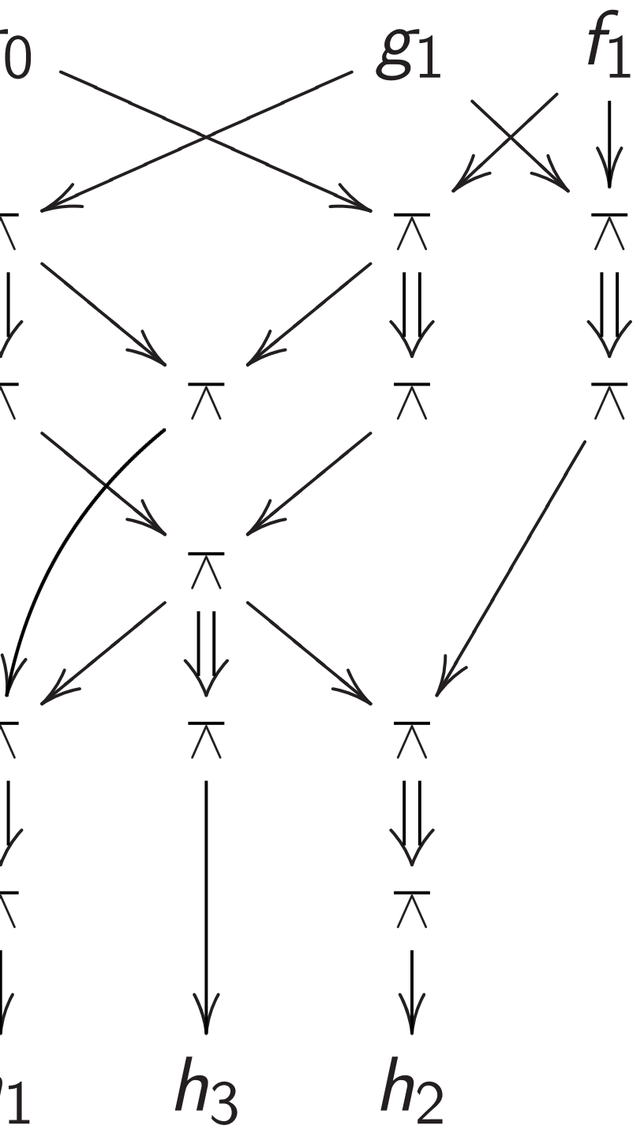

: $a, b \mapsto 1 - ab$ computing

$h_0 + 2h_1 + 4h_2 + 8h_3$

ers $f_0 + 2f_1, g_0 + 2g_1$.

Electricity takes time to percolate through wires and gates.
If f_0, f_1, g_0, g_1 are stable
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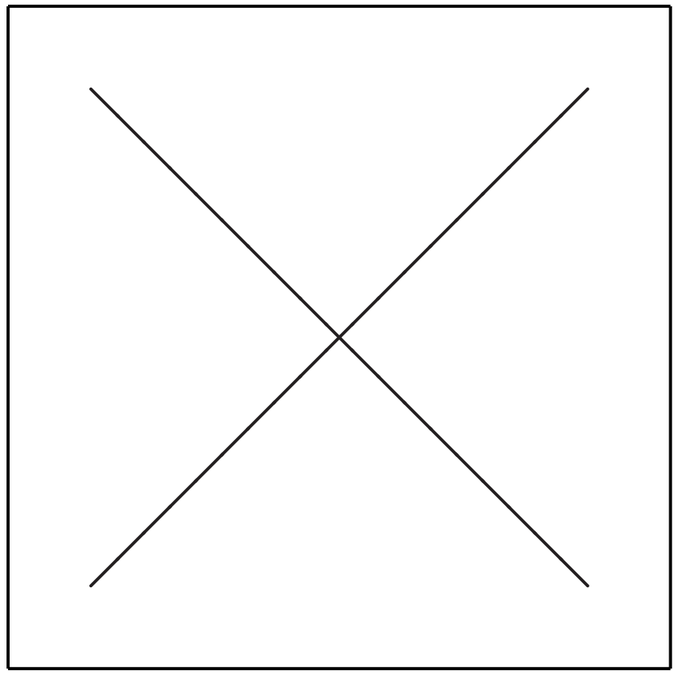

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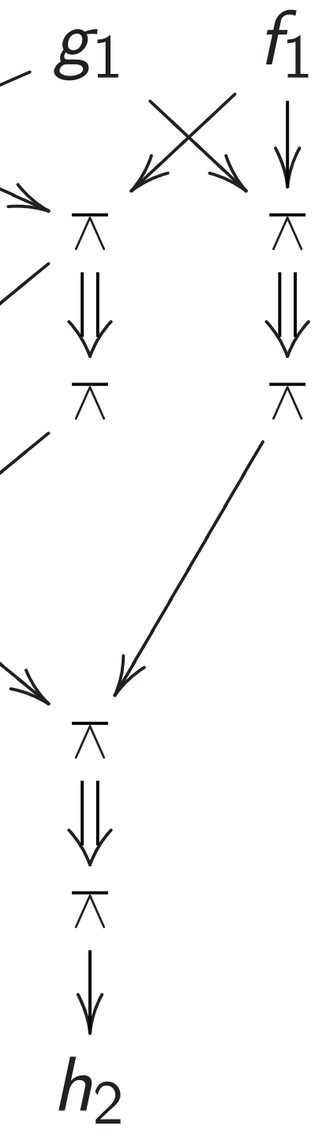

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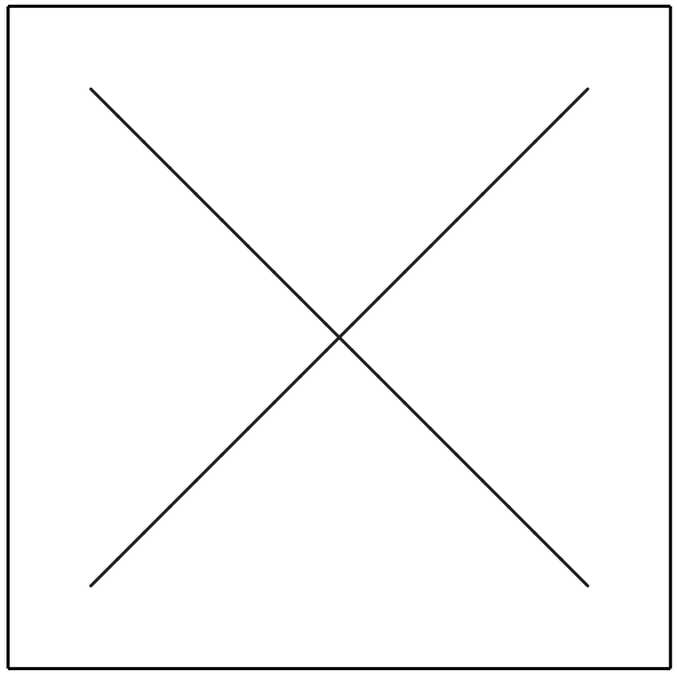

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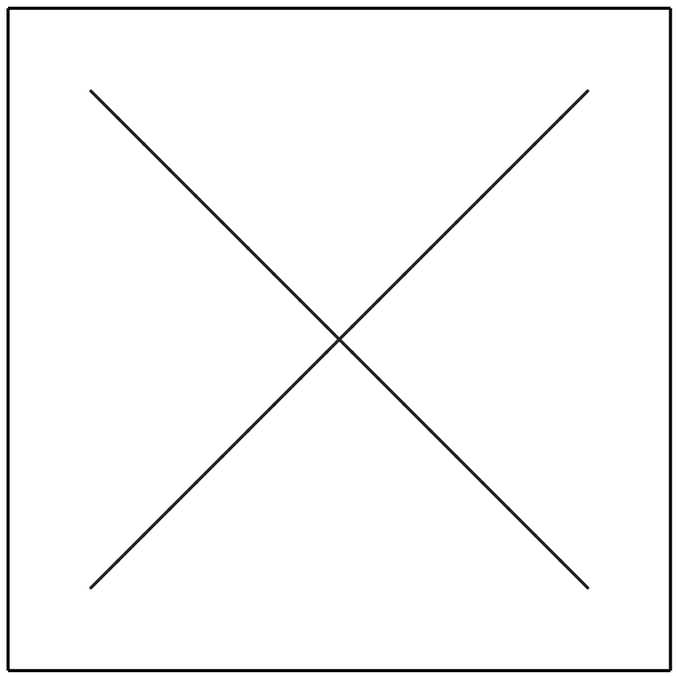

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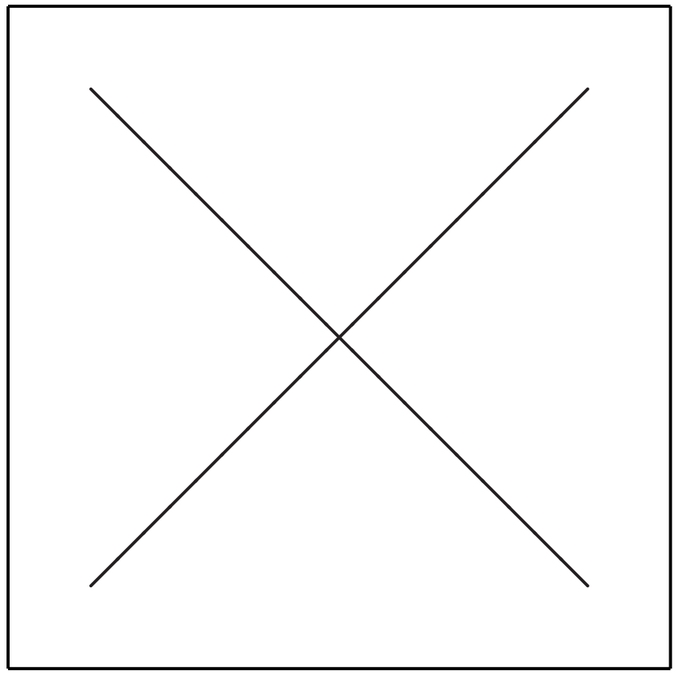

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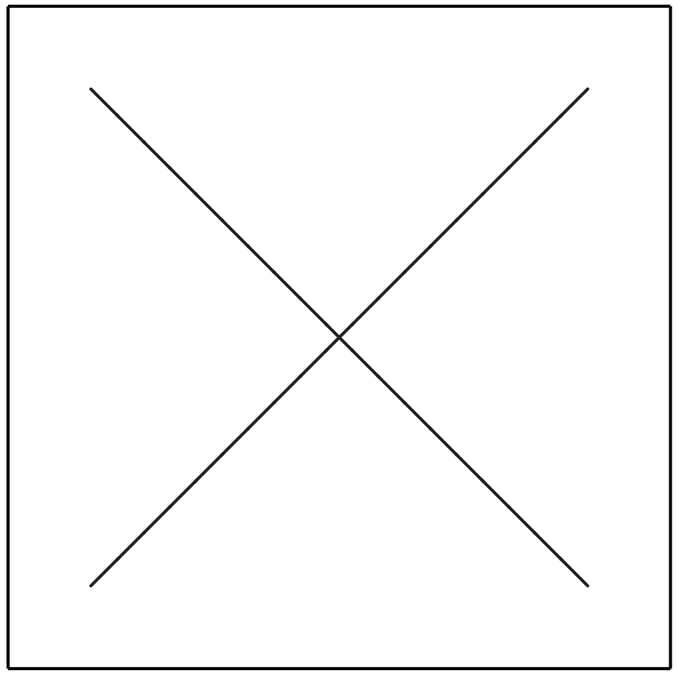

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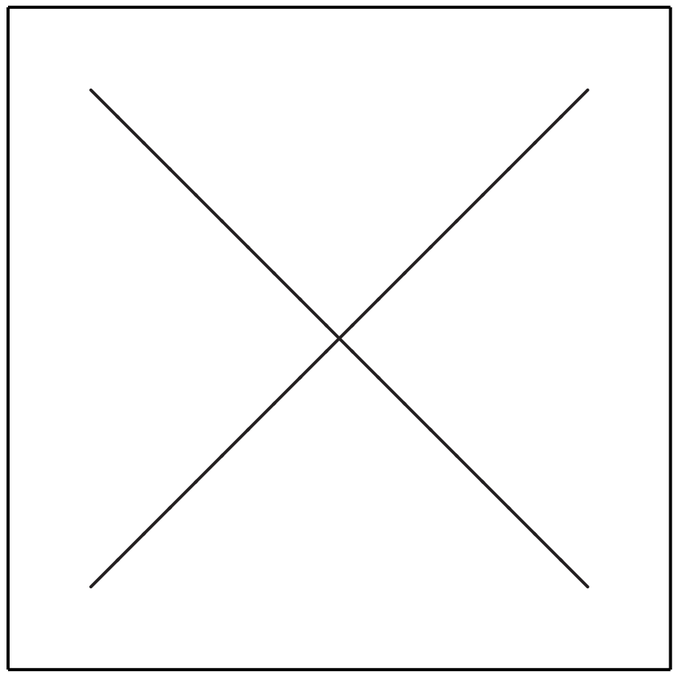

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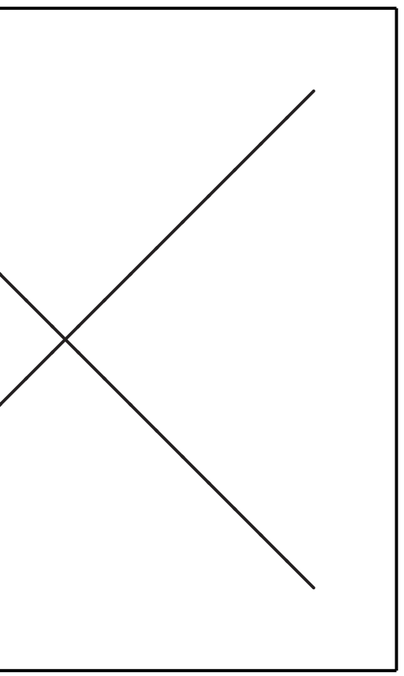

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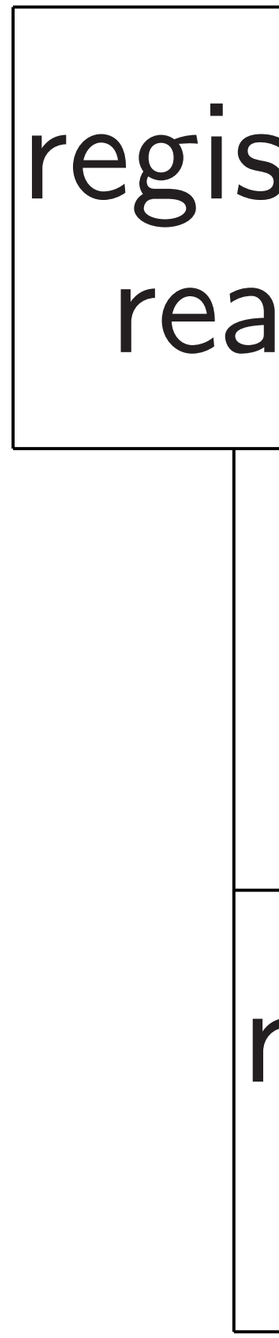

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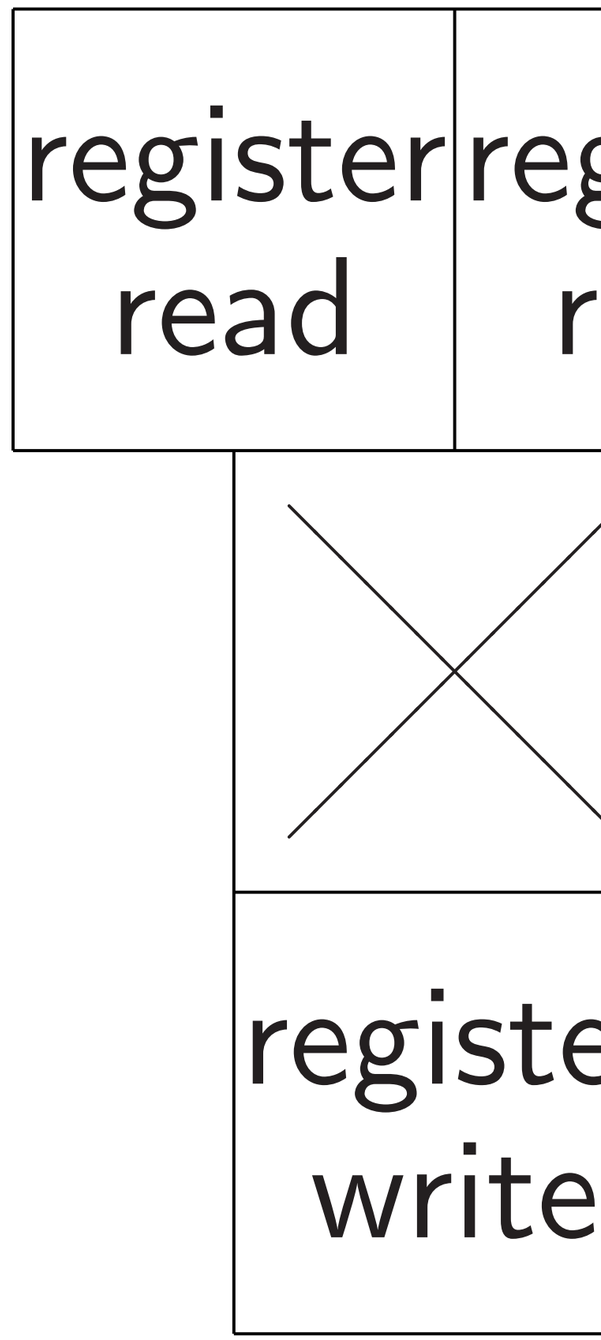

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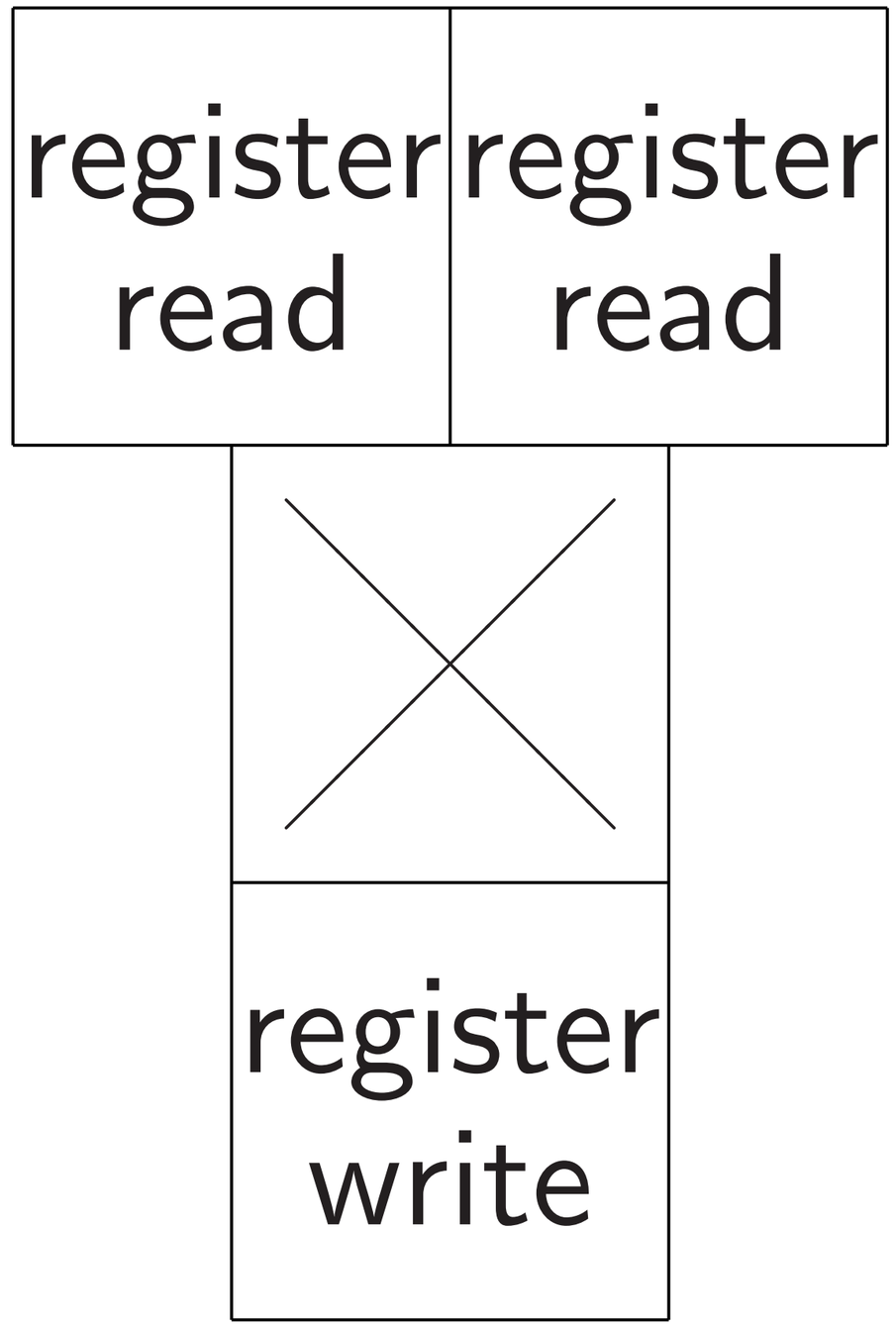

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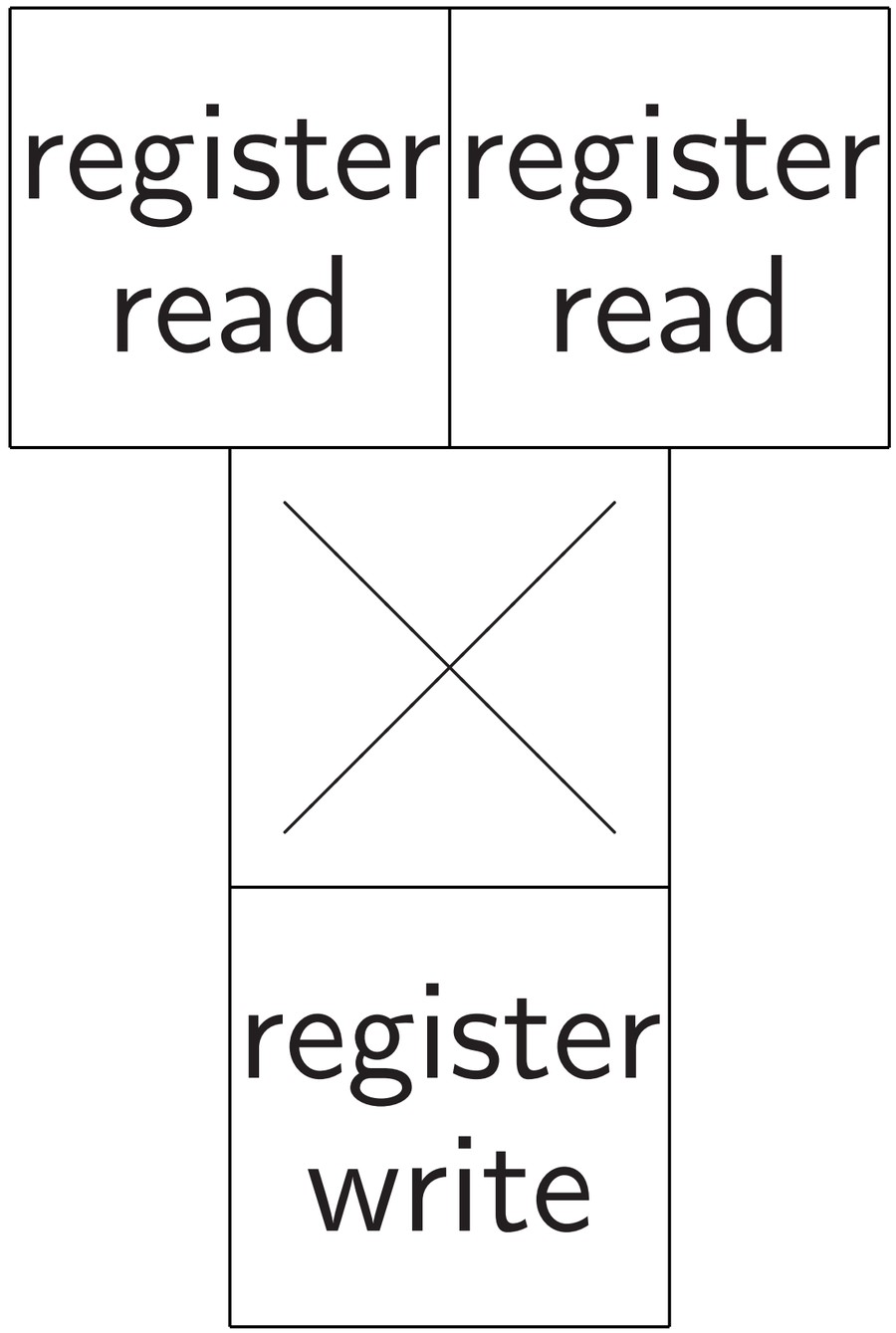

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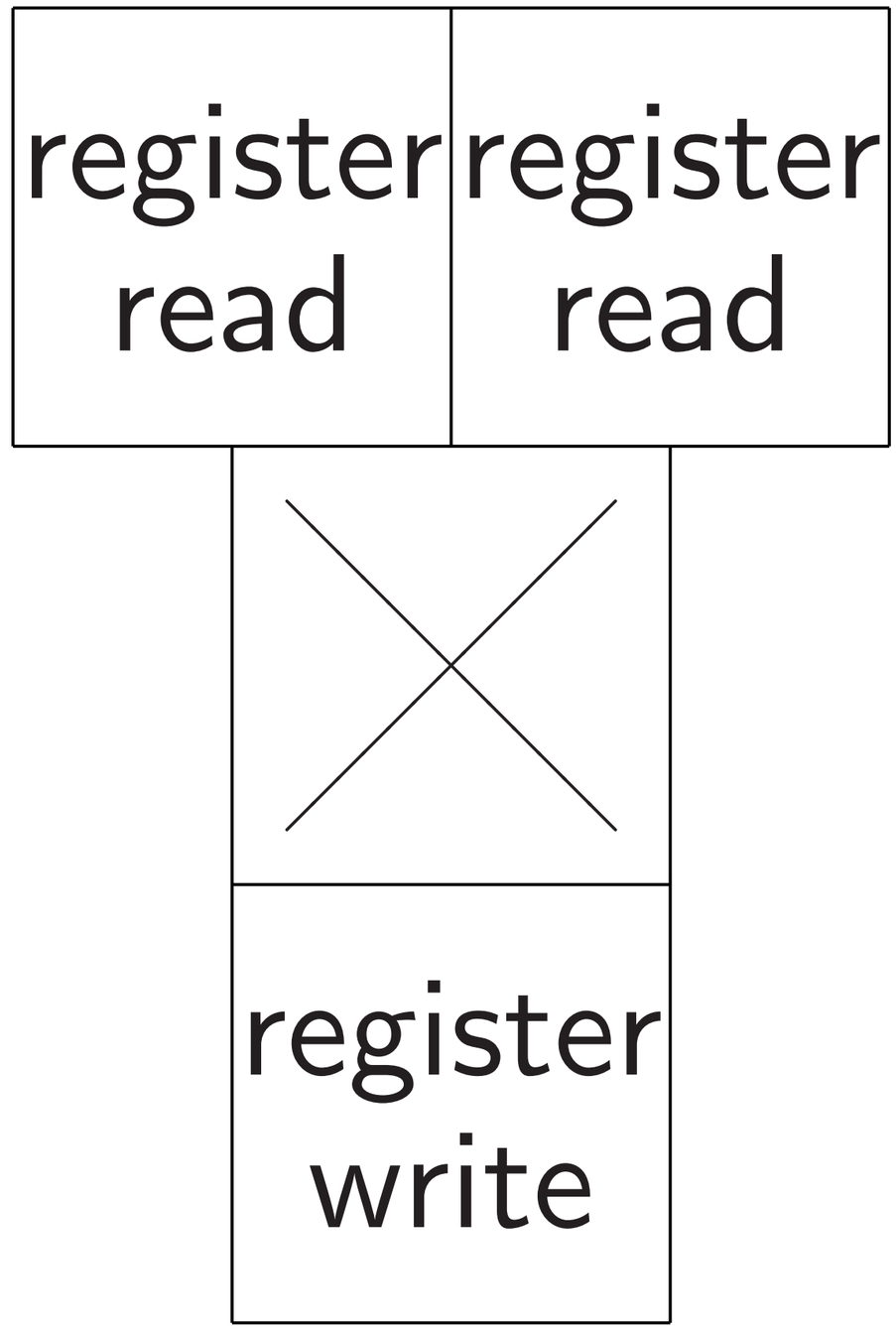

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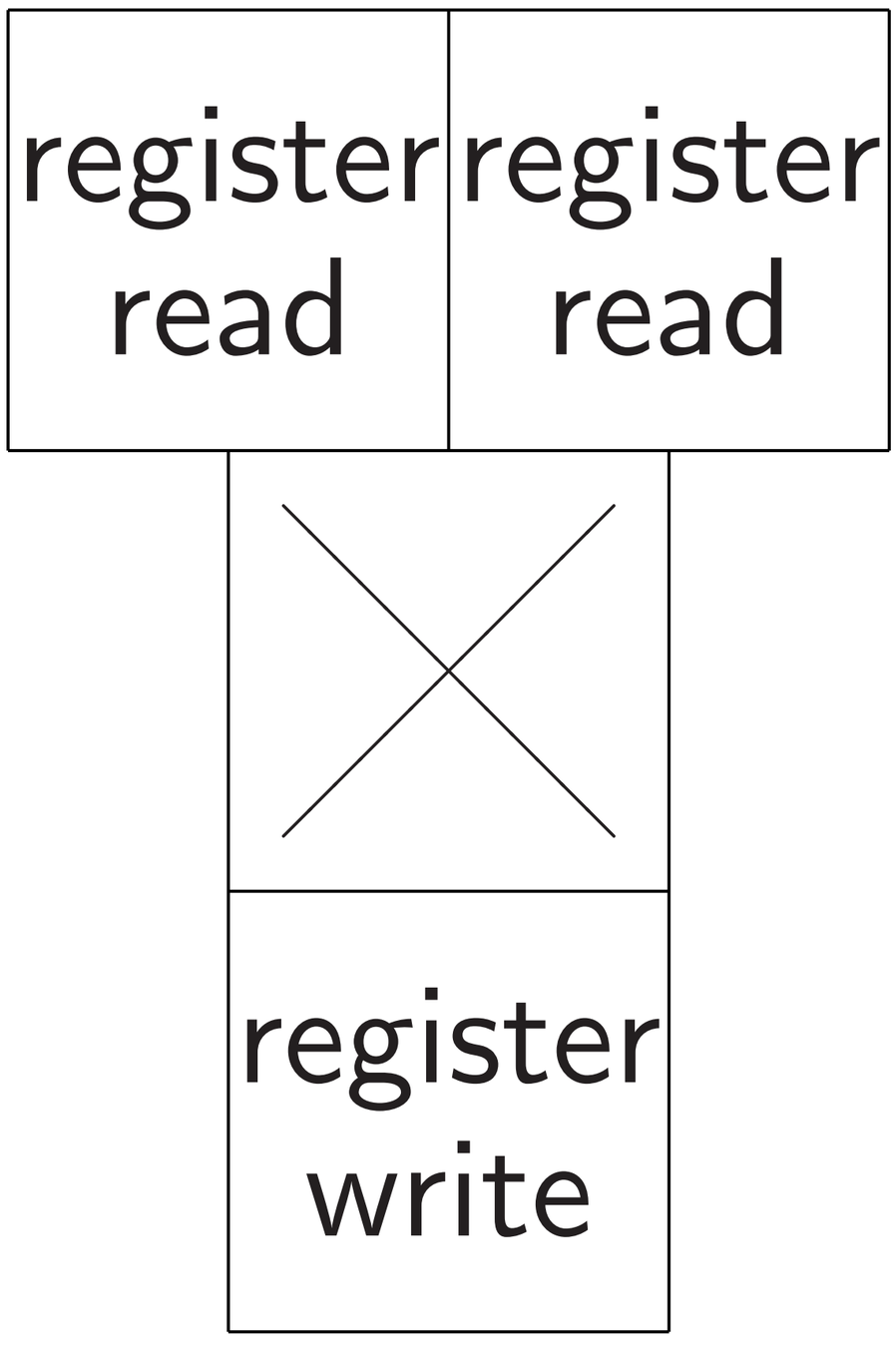

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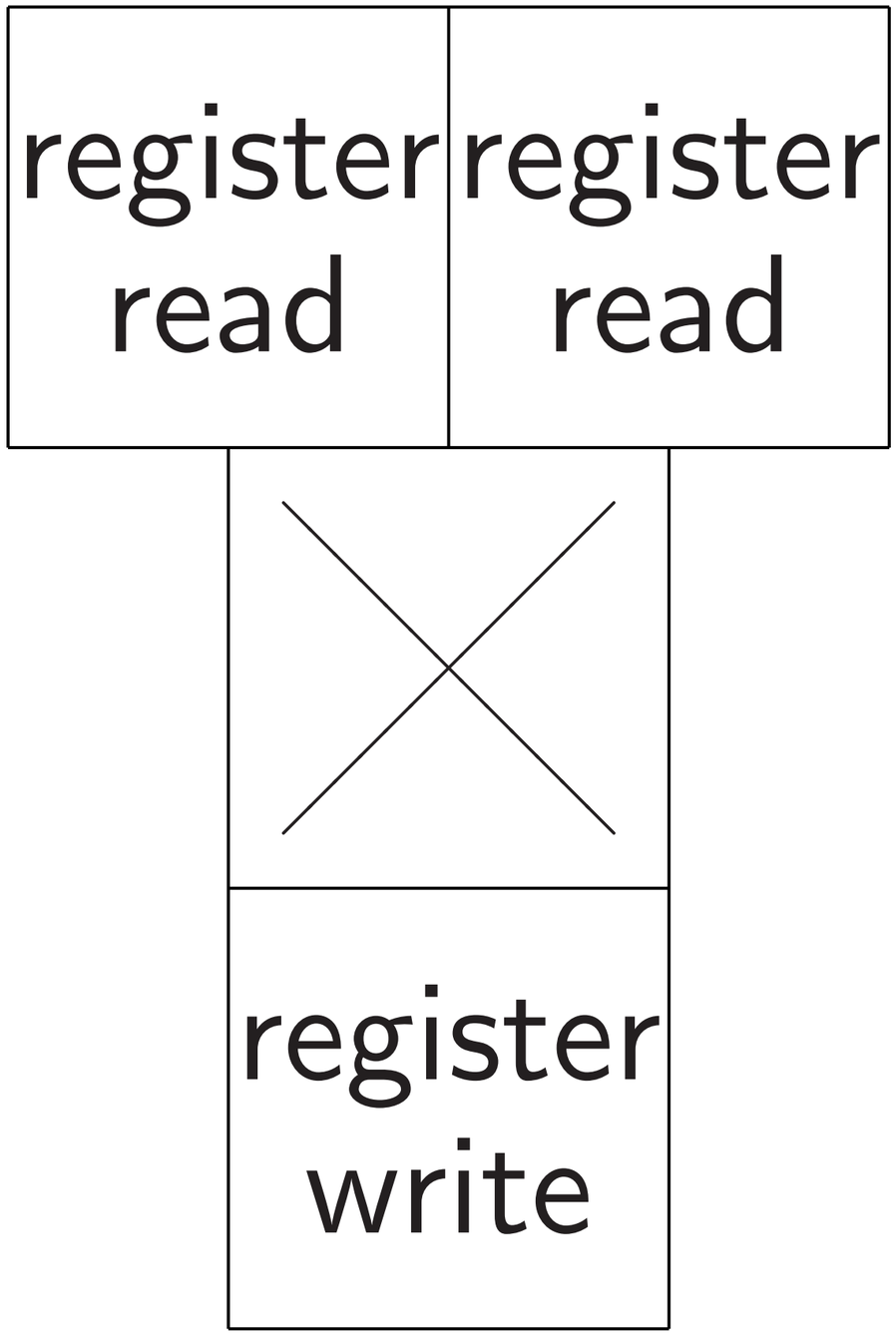

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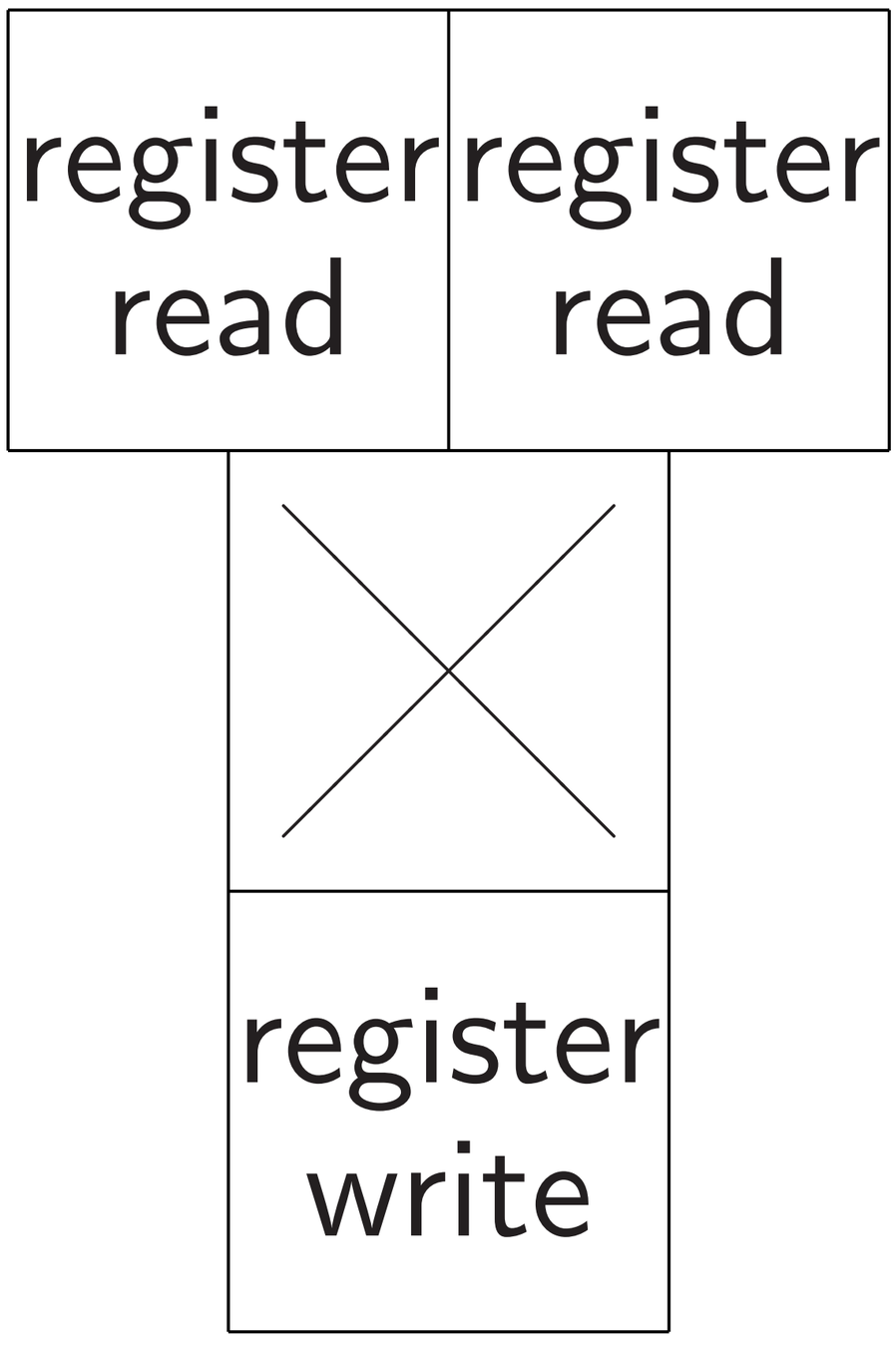

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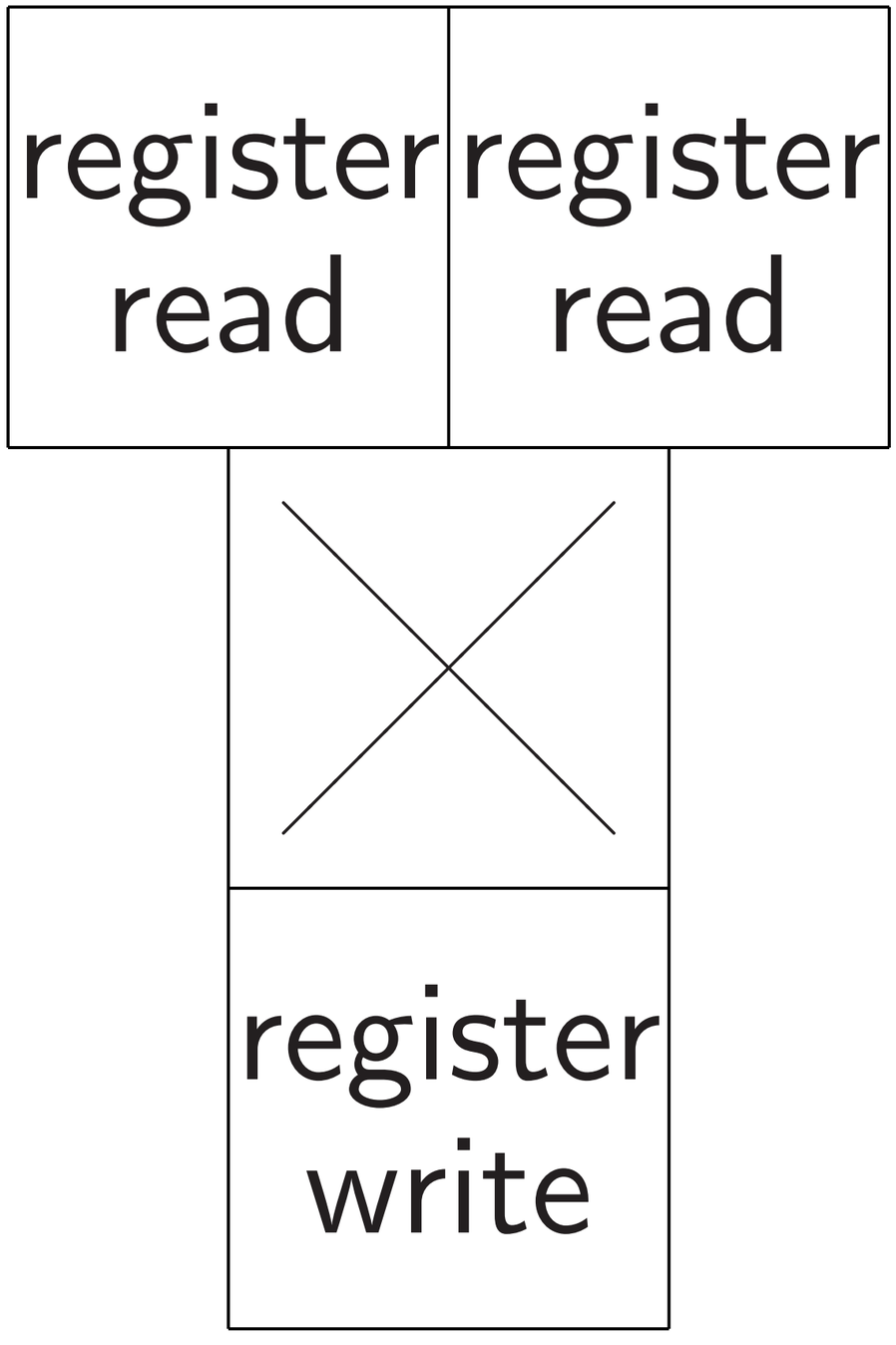

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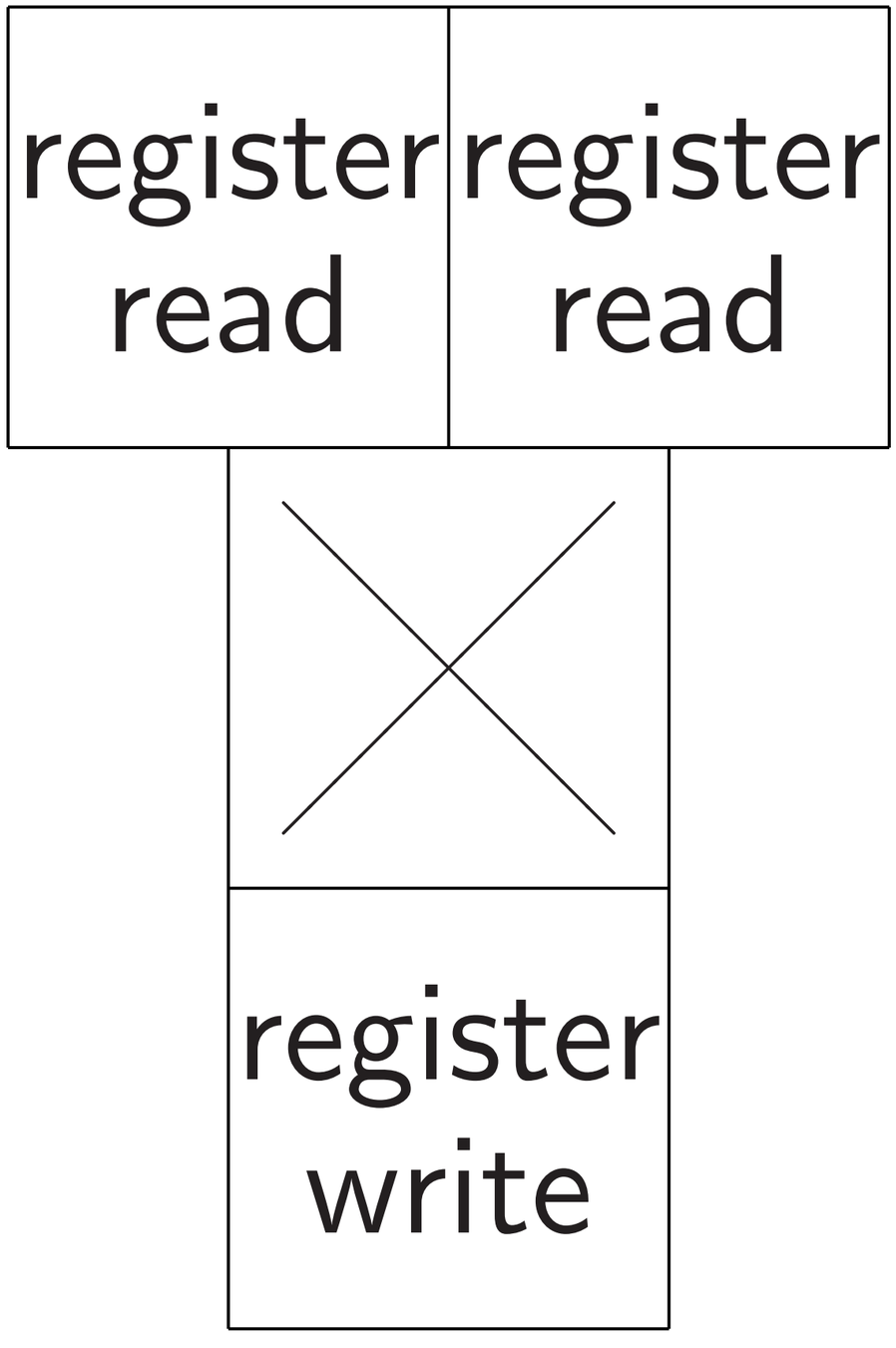

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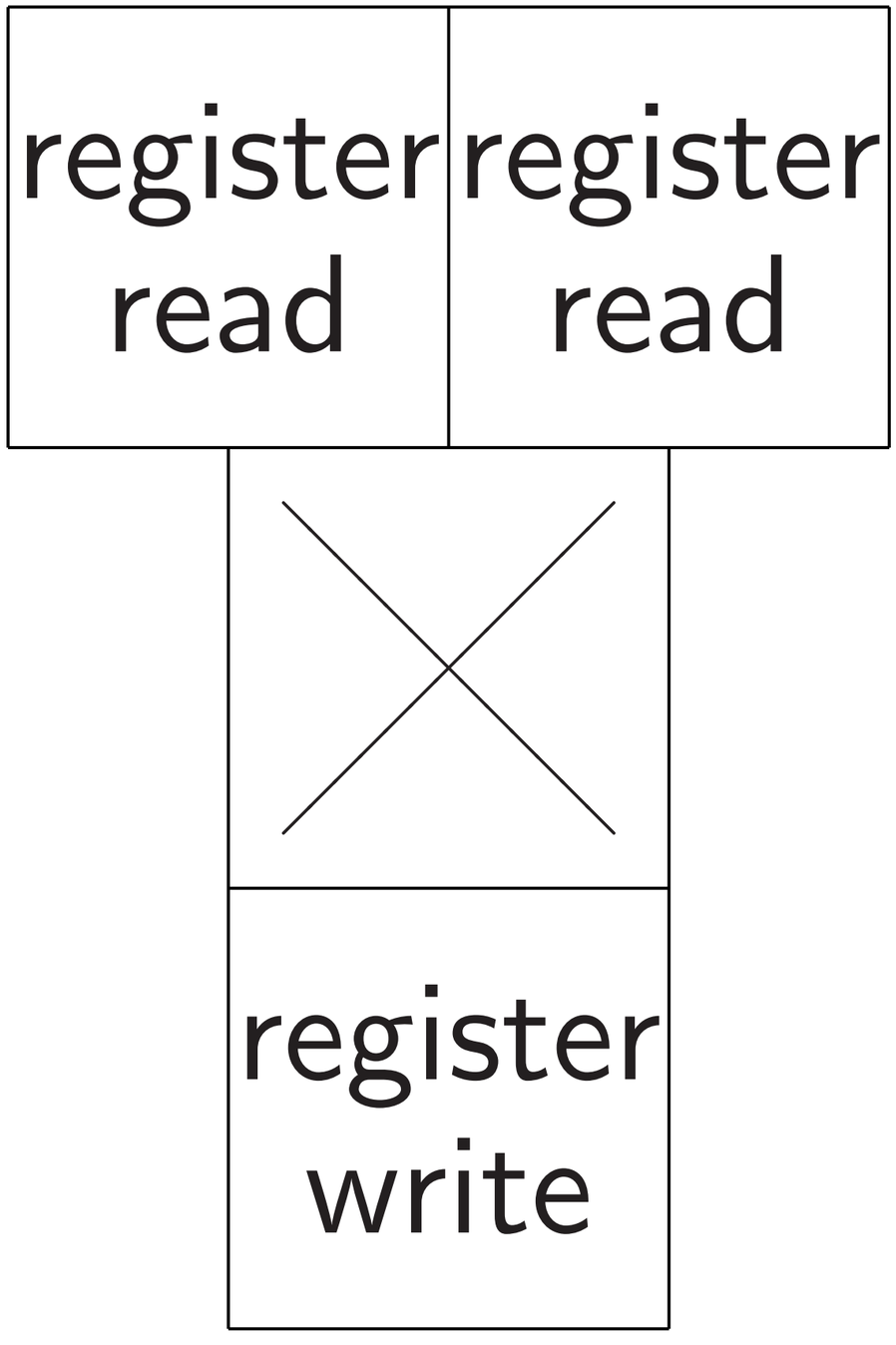

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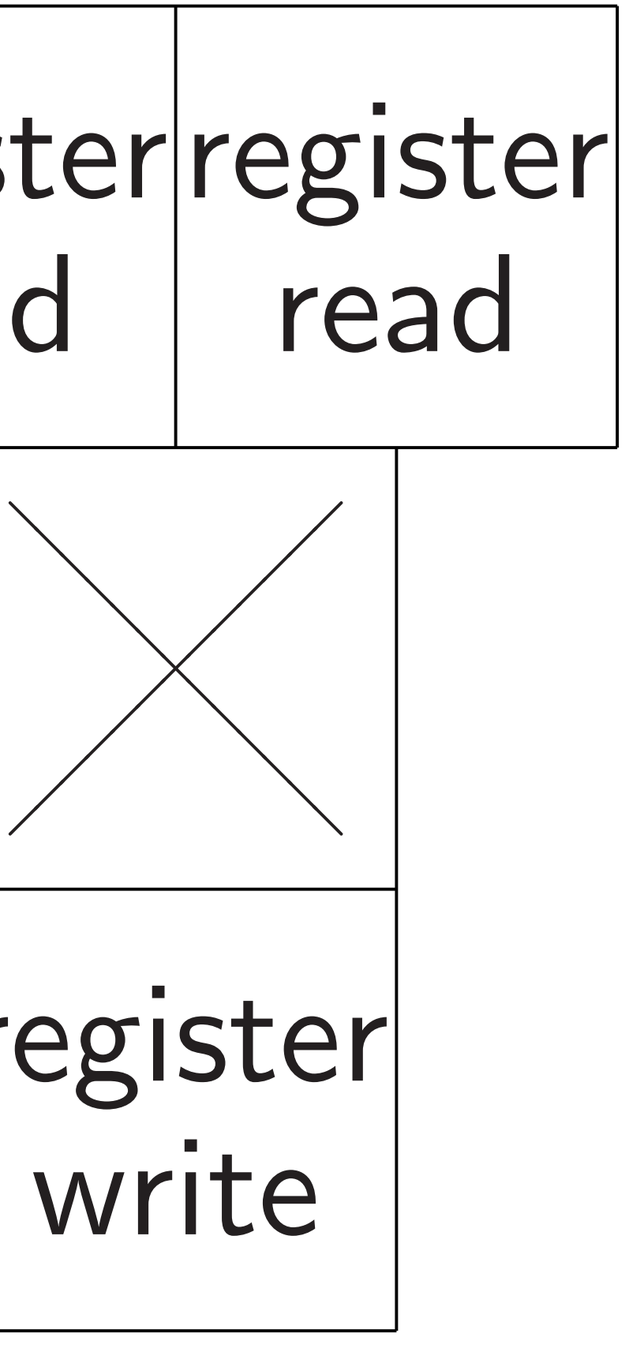

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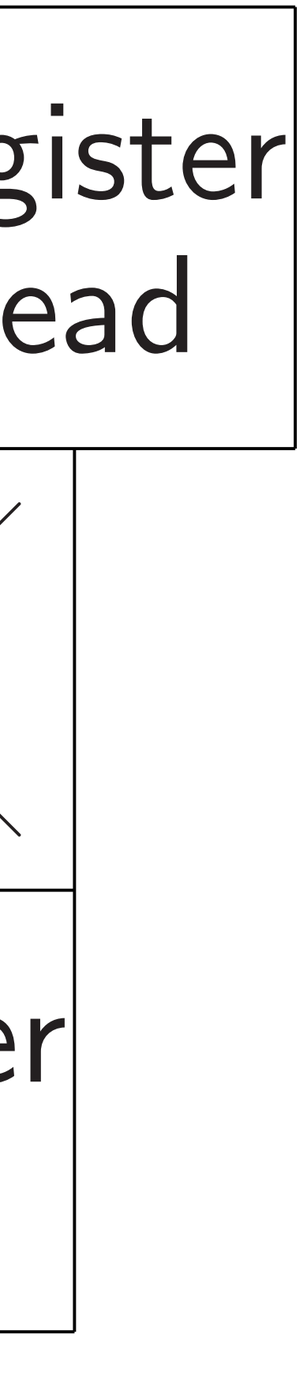

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Hook $(p, r_0, \ldots, r_{15})$
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Hook outputs $(p', r'_0, \ldots, r'_{15}$
into the same flip-flops.

At each “clock tick”,
flip-flops are overwritten
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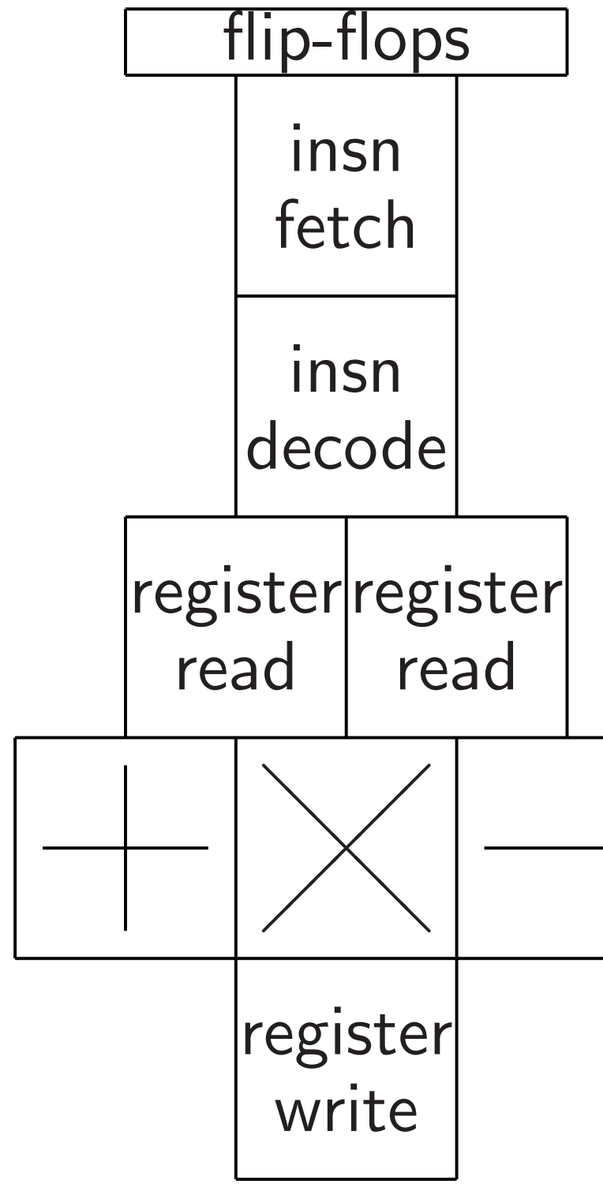

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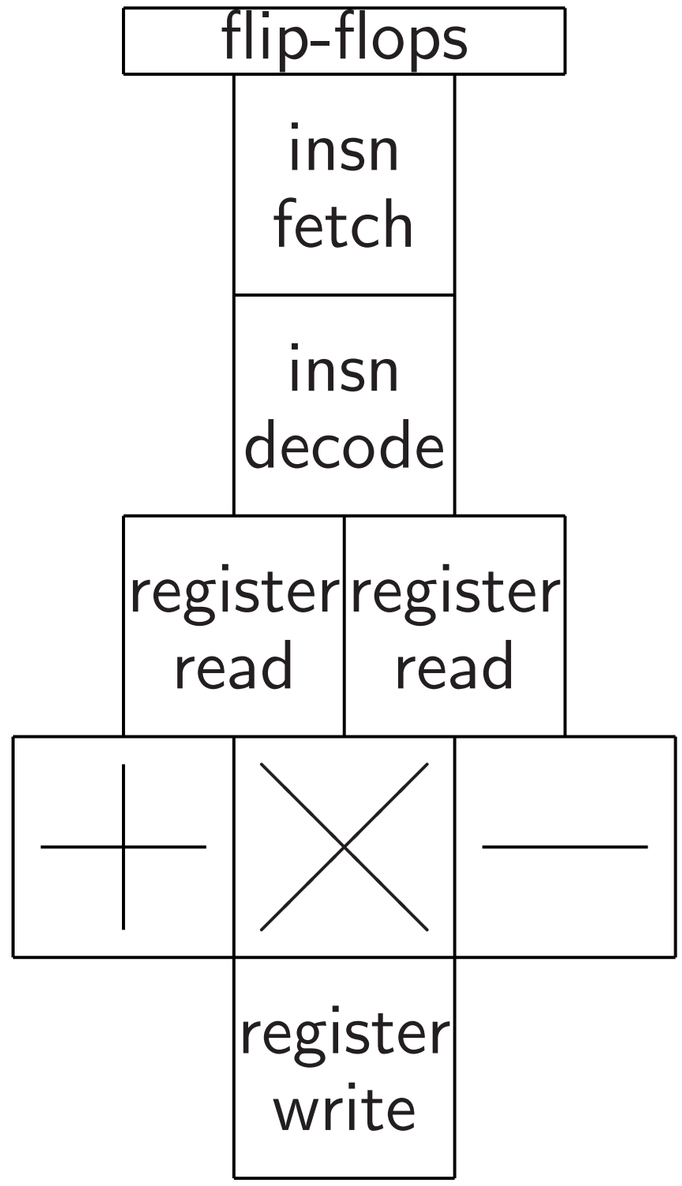
Now have semi-flexible CPU



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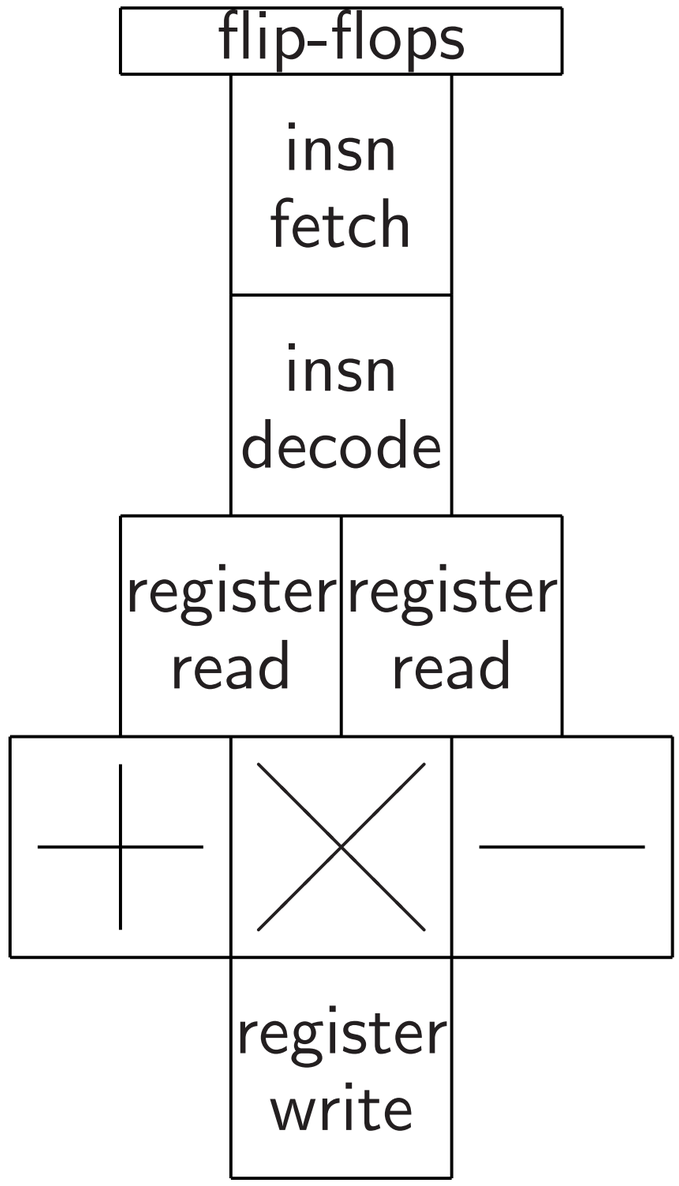
from flip-flops to flip-flops.

Now have semi-flexible CPU:



Further flexibility is useful:
e.g., logic instructions.

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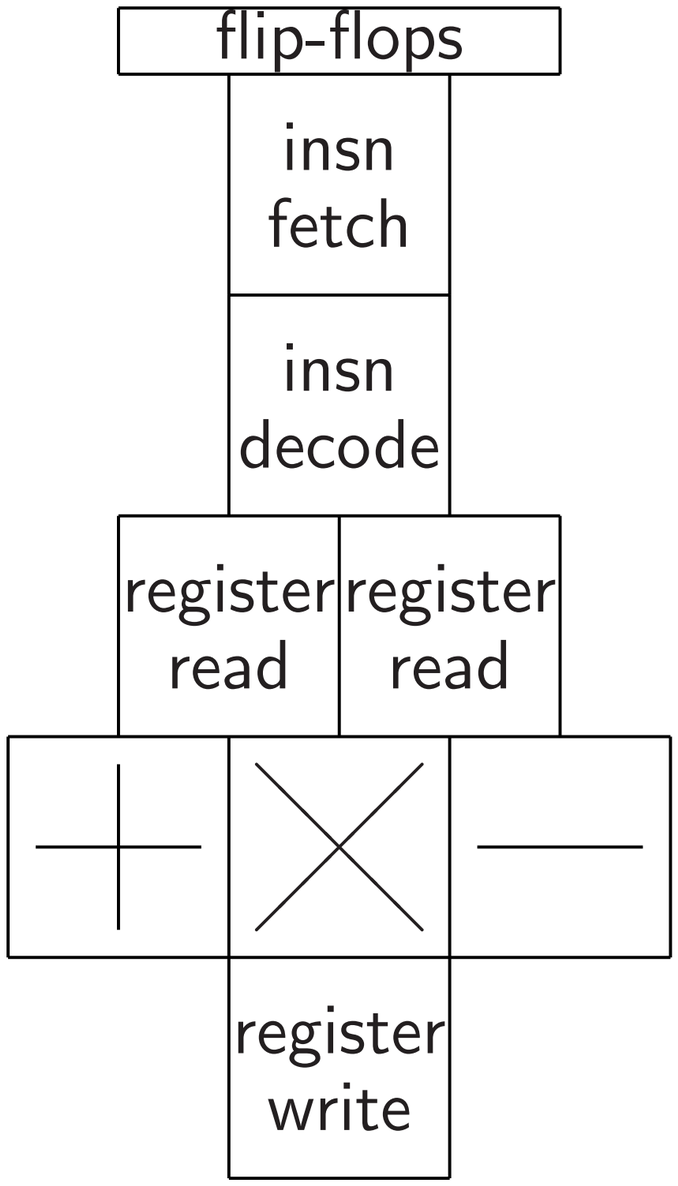

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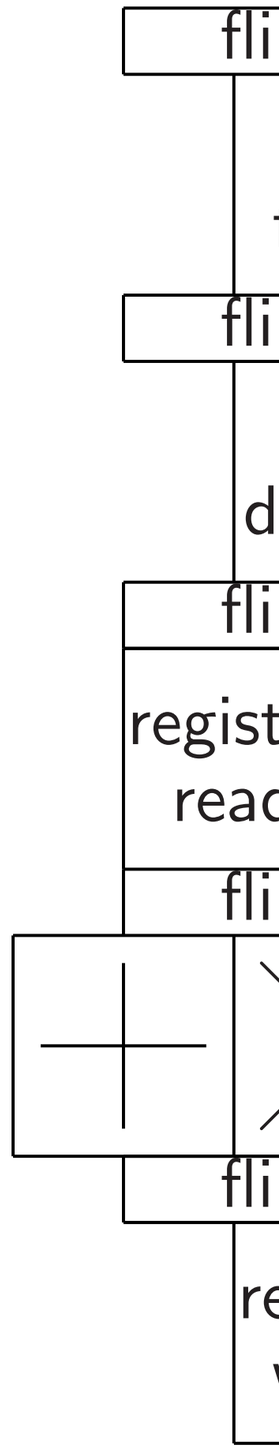

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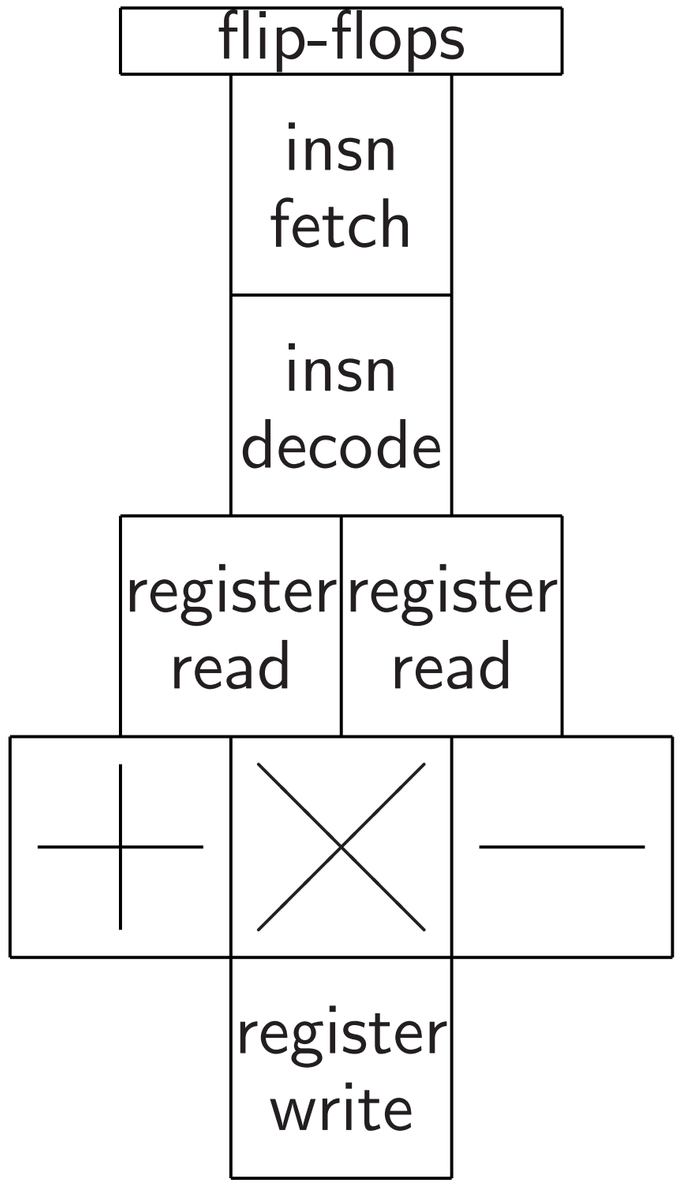

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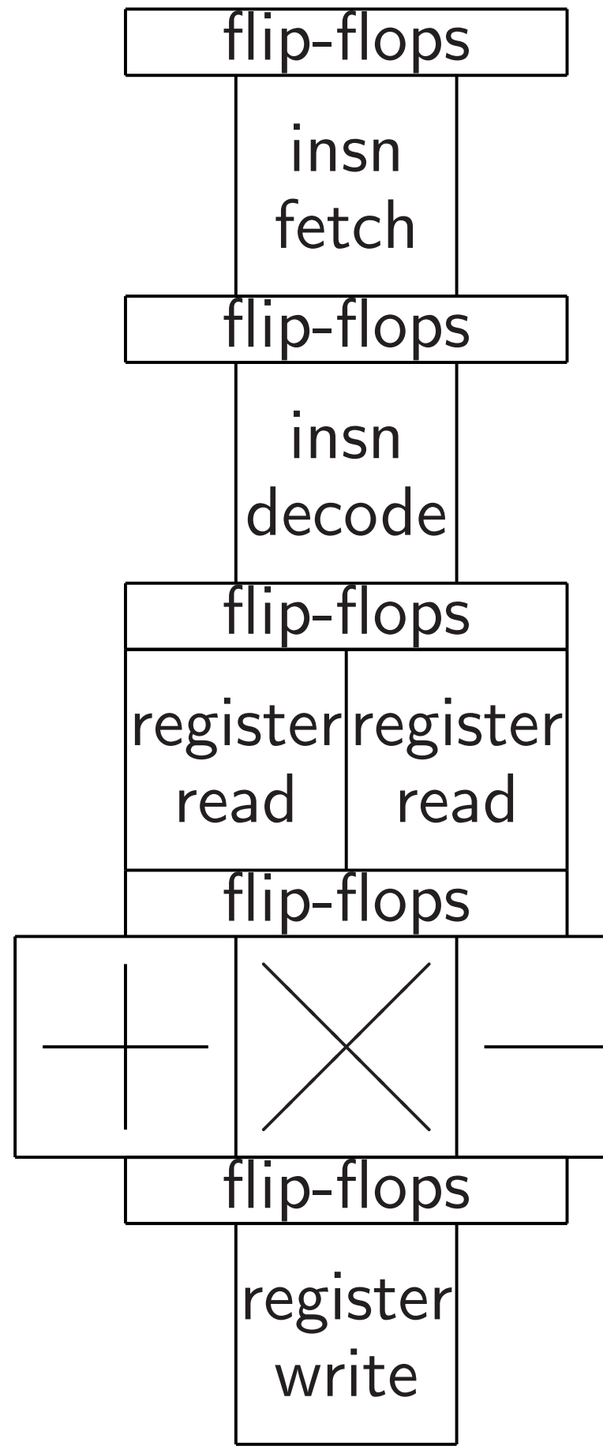

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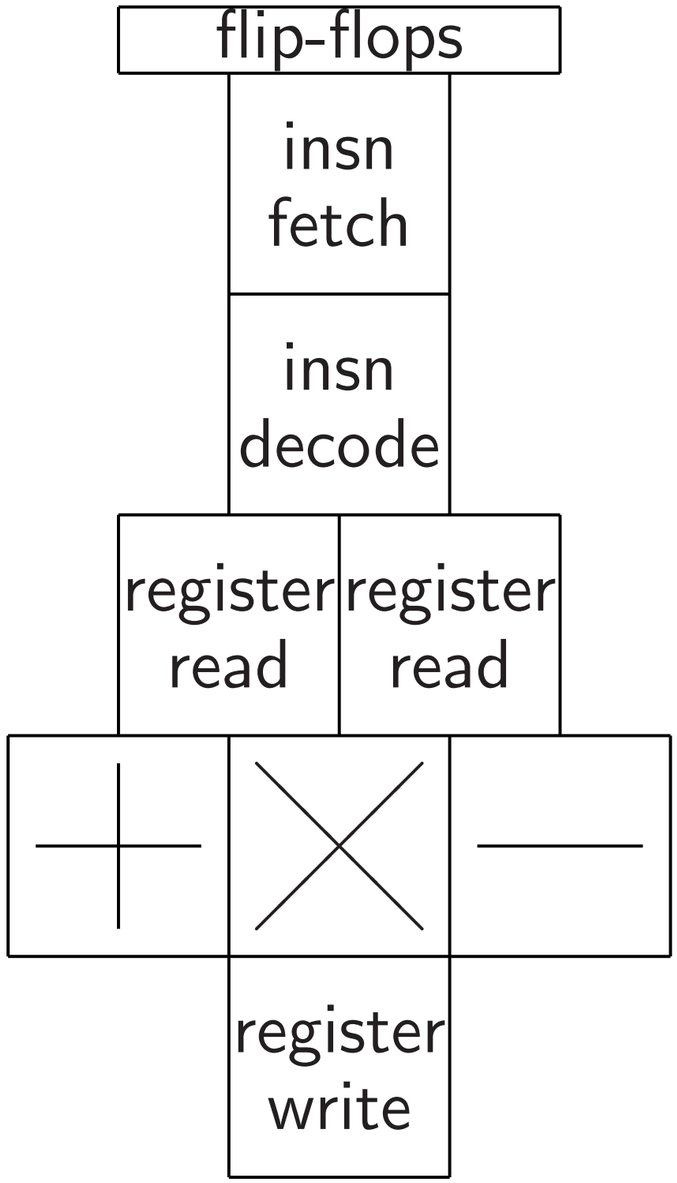

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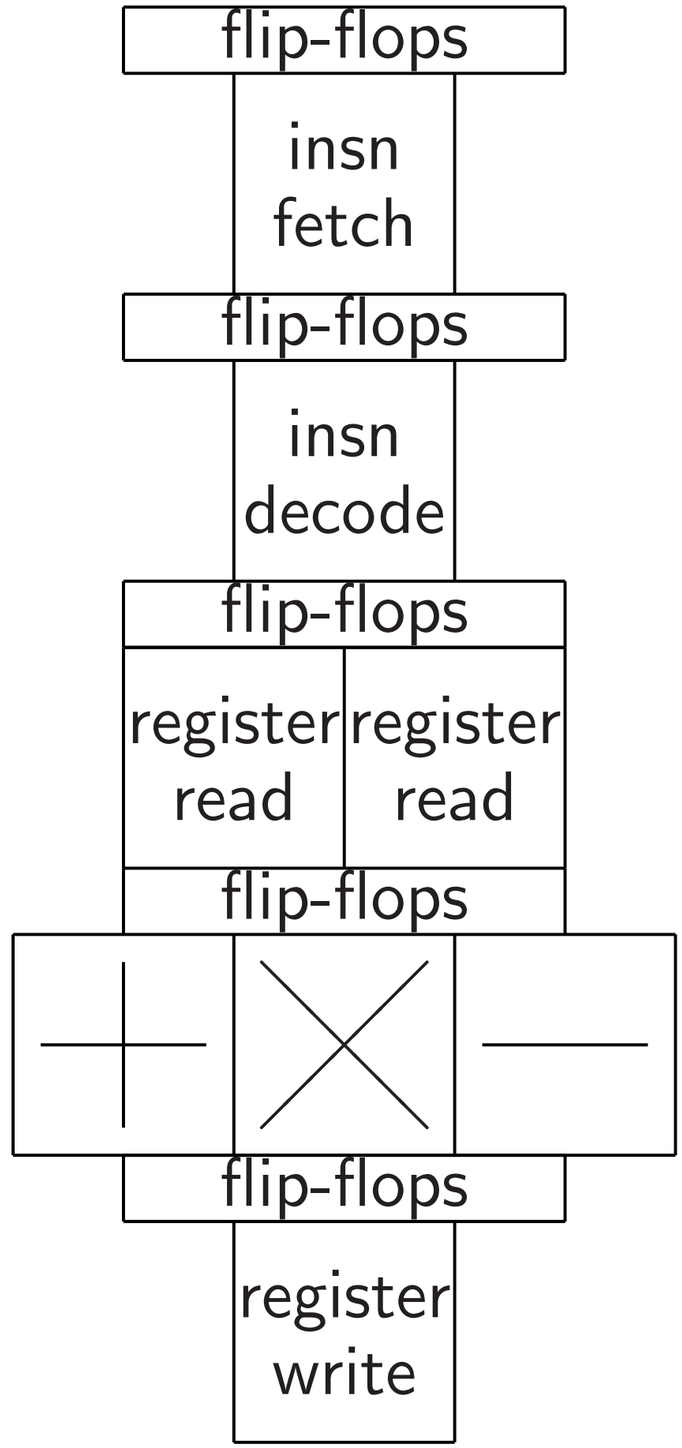
"Pipelining" allows faster cl



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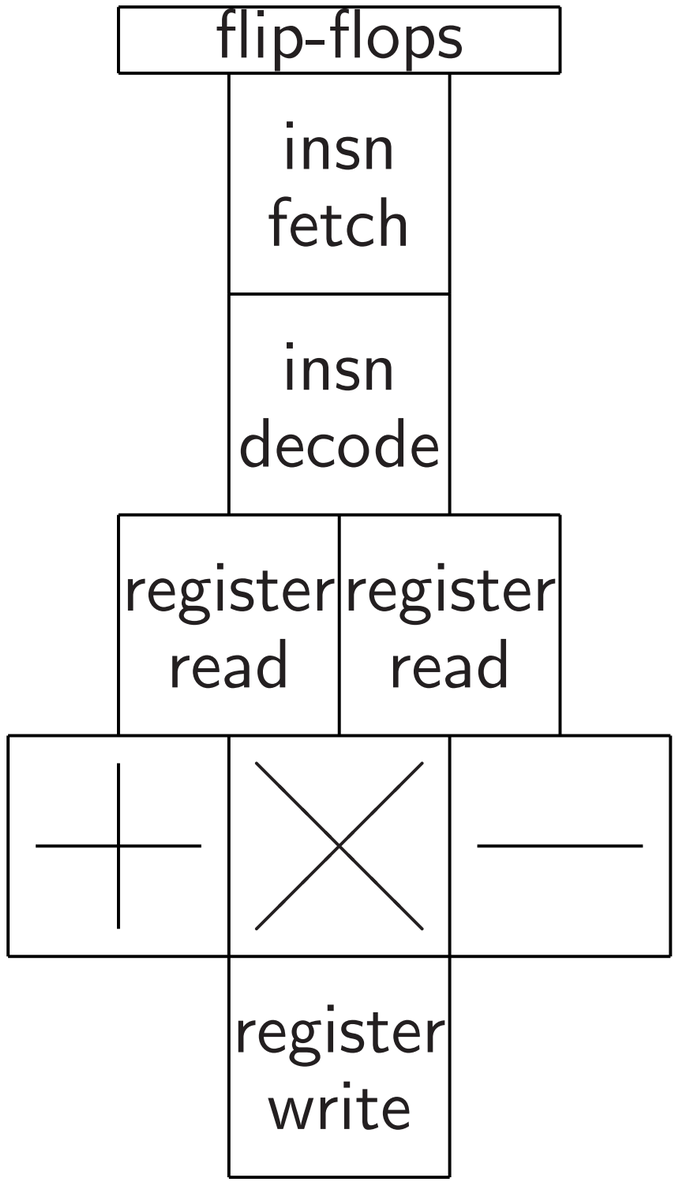

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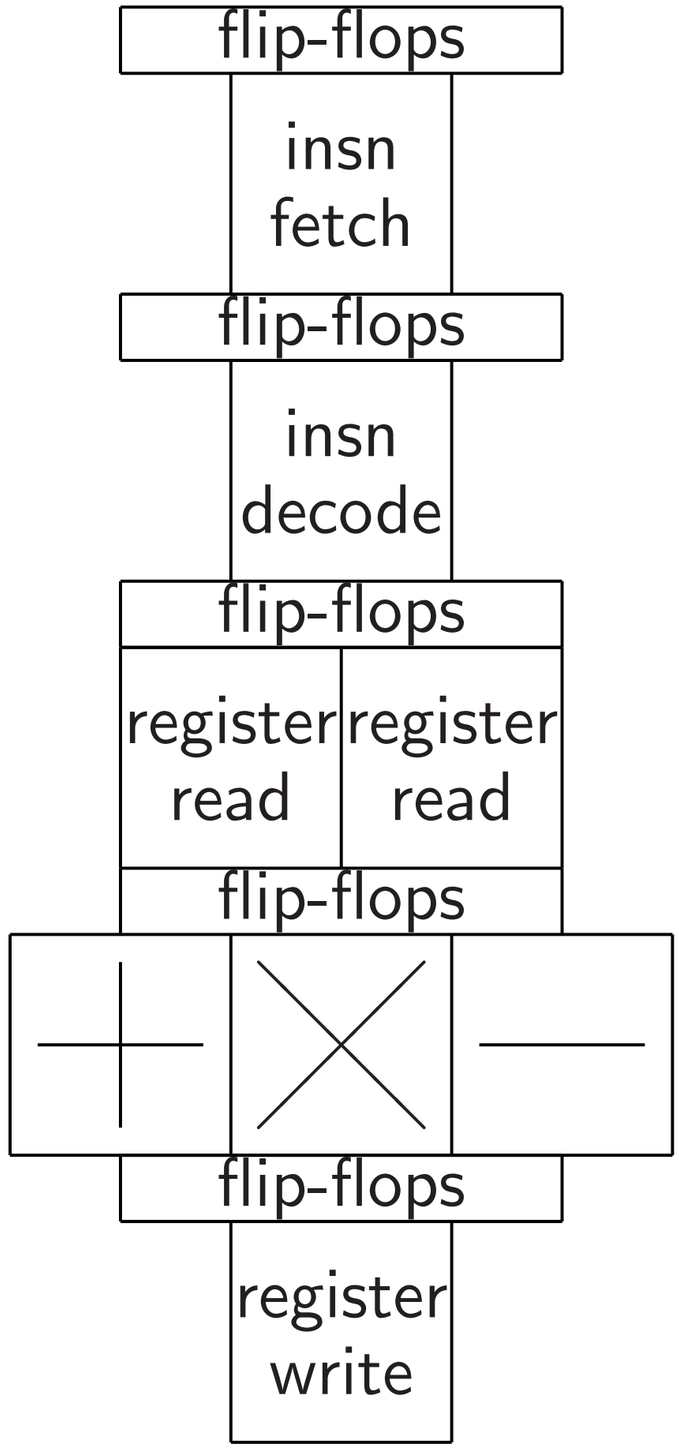

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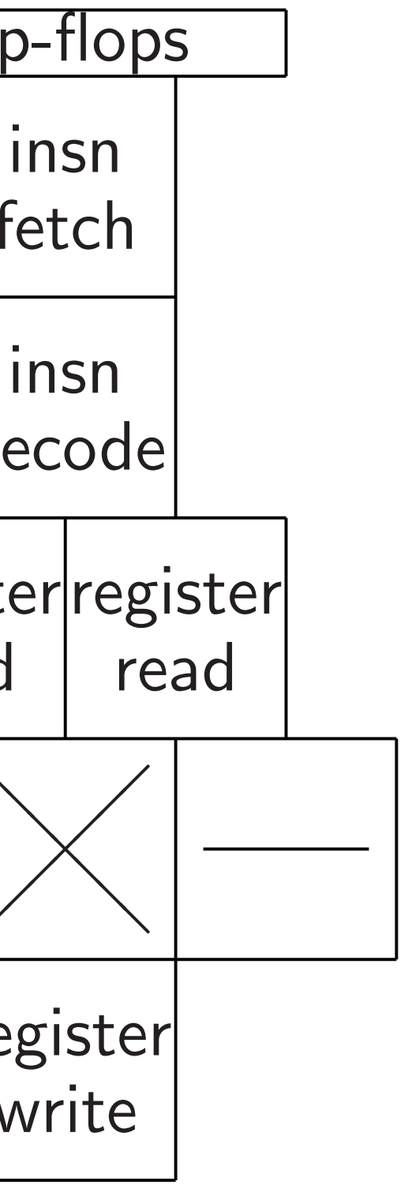

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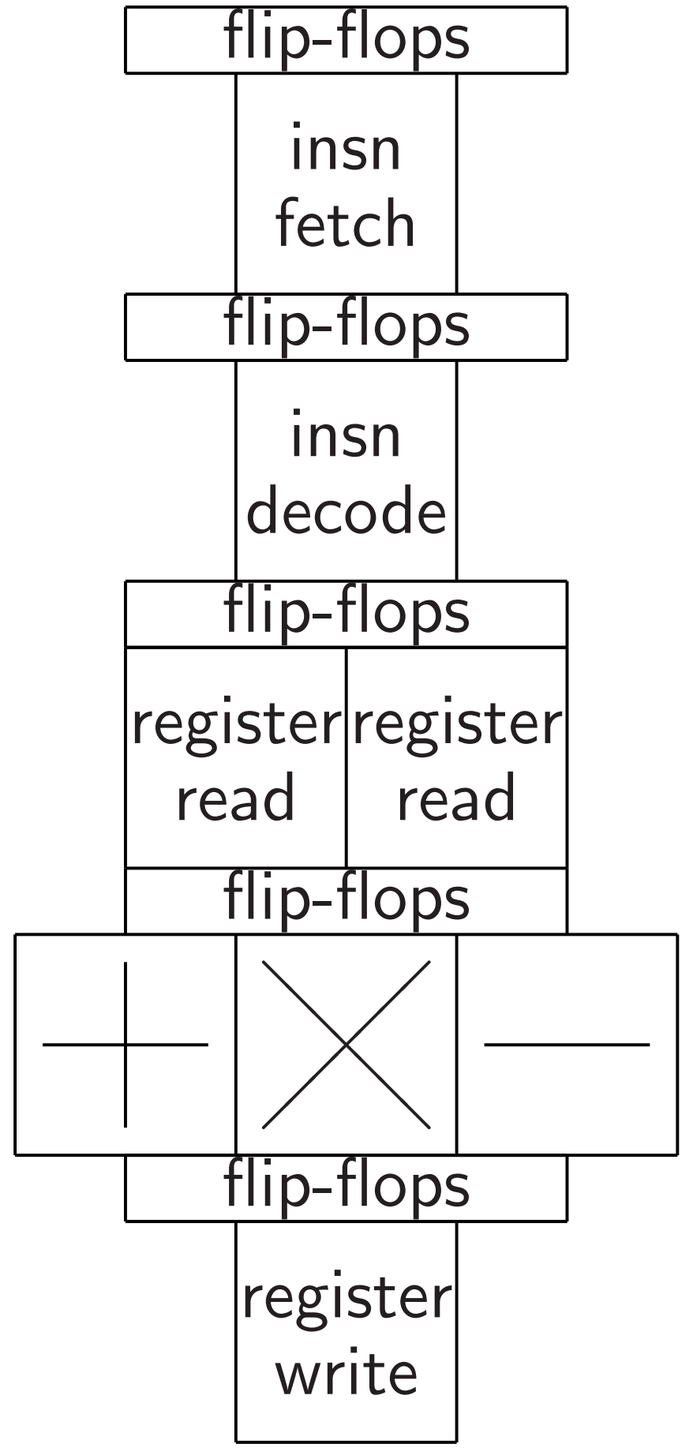

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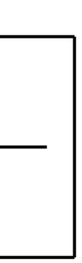

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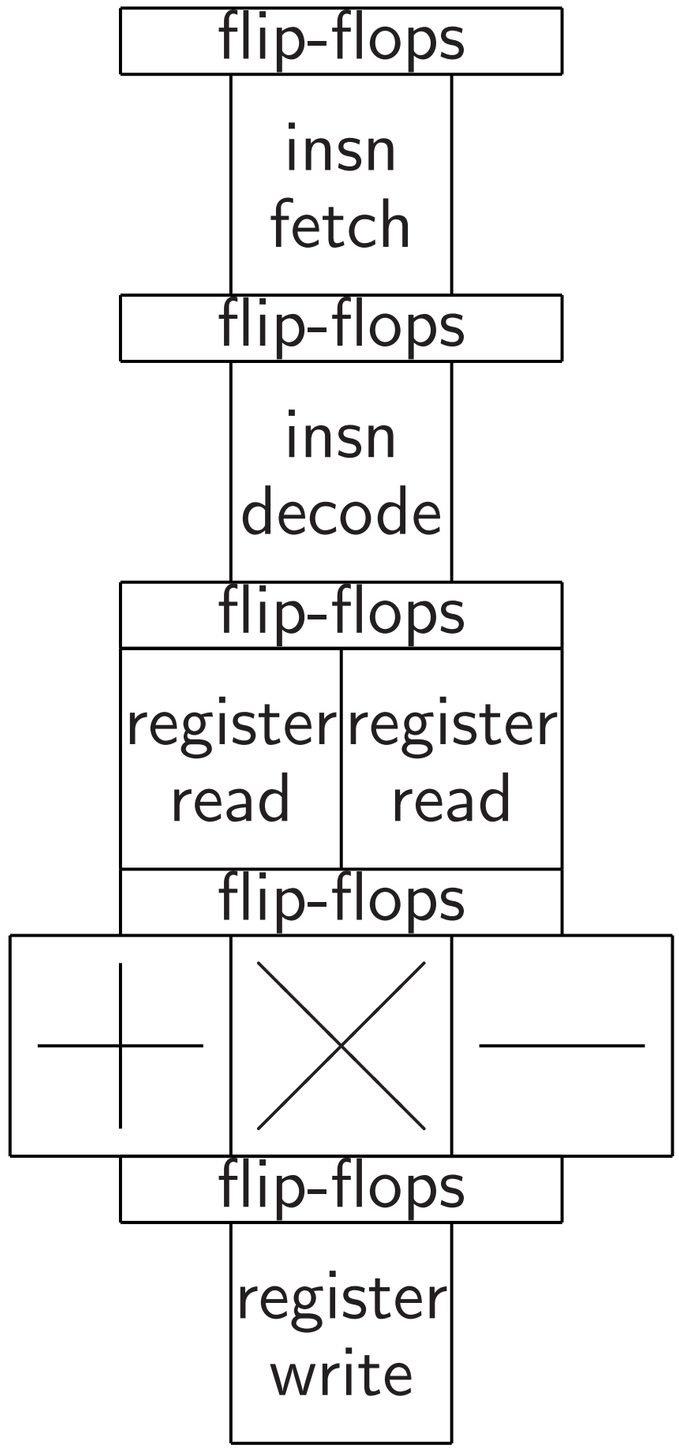

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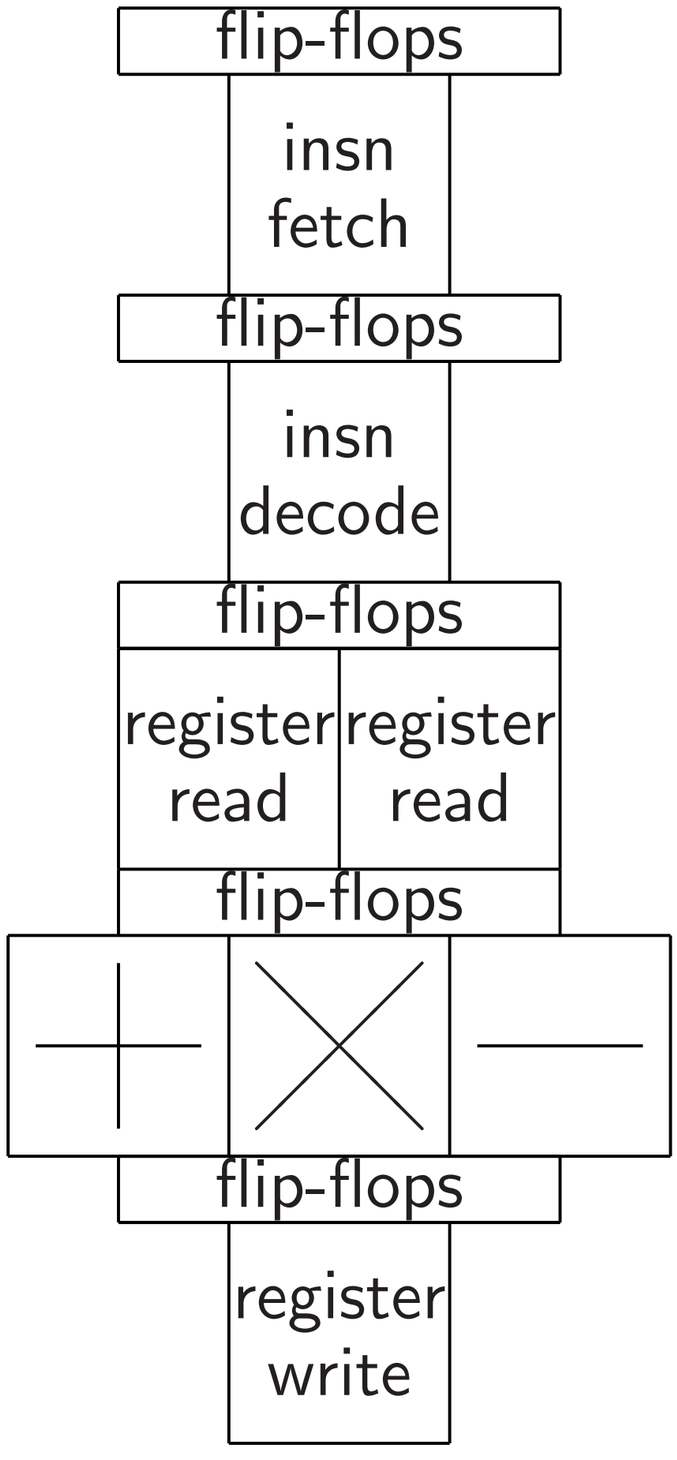

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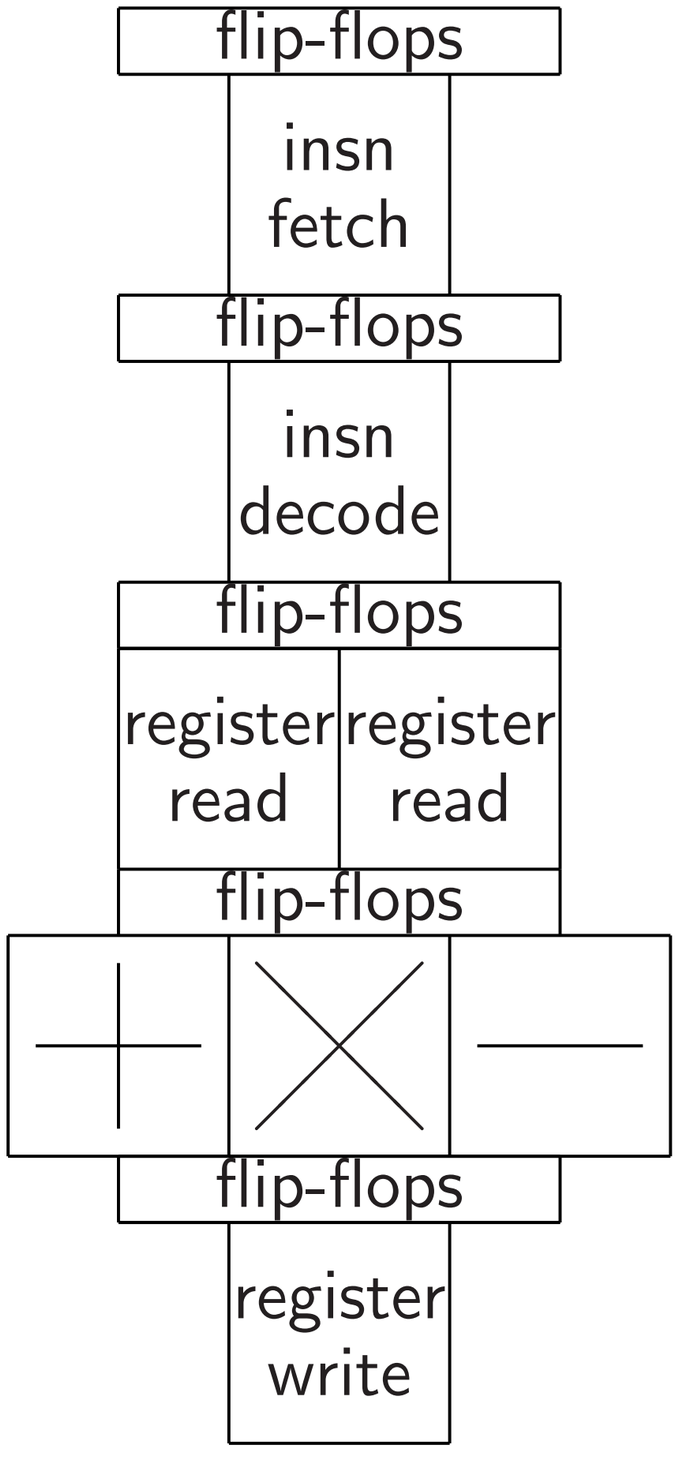

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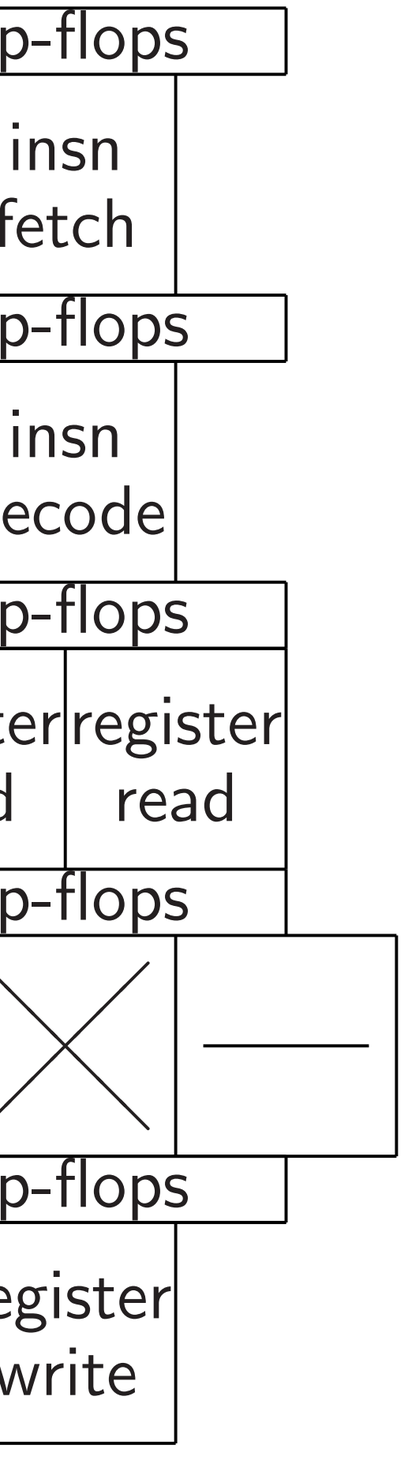

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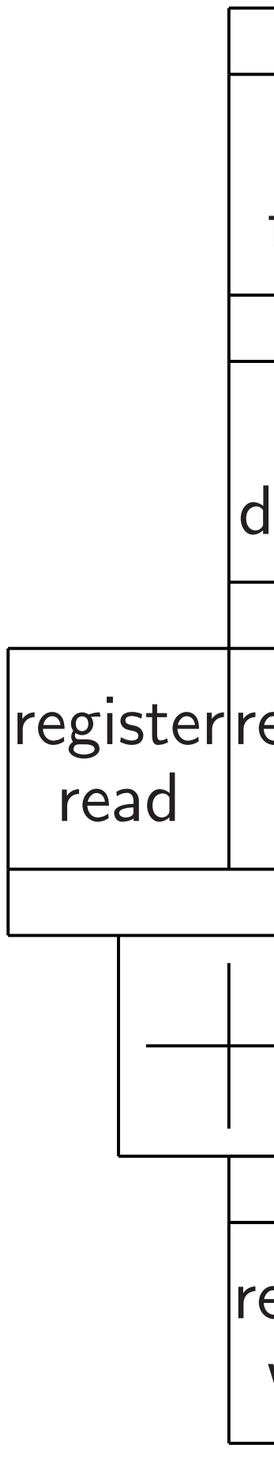

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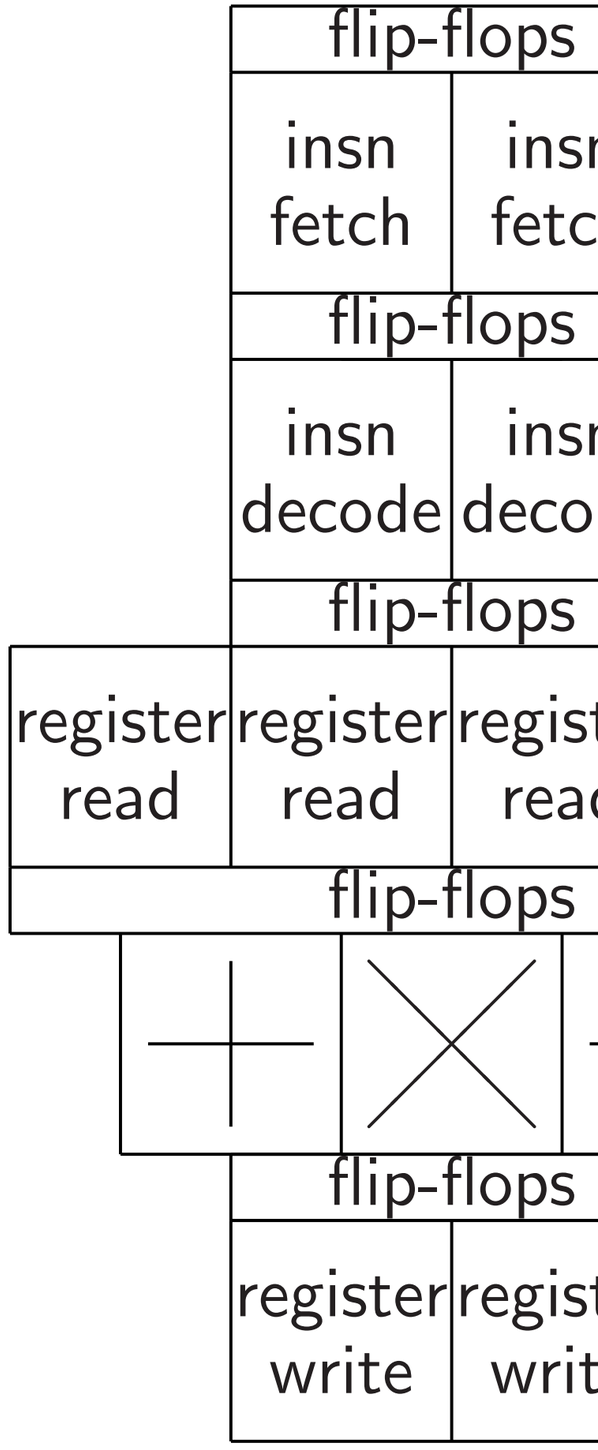

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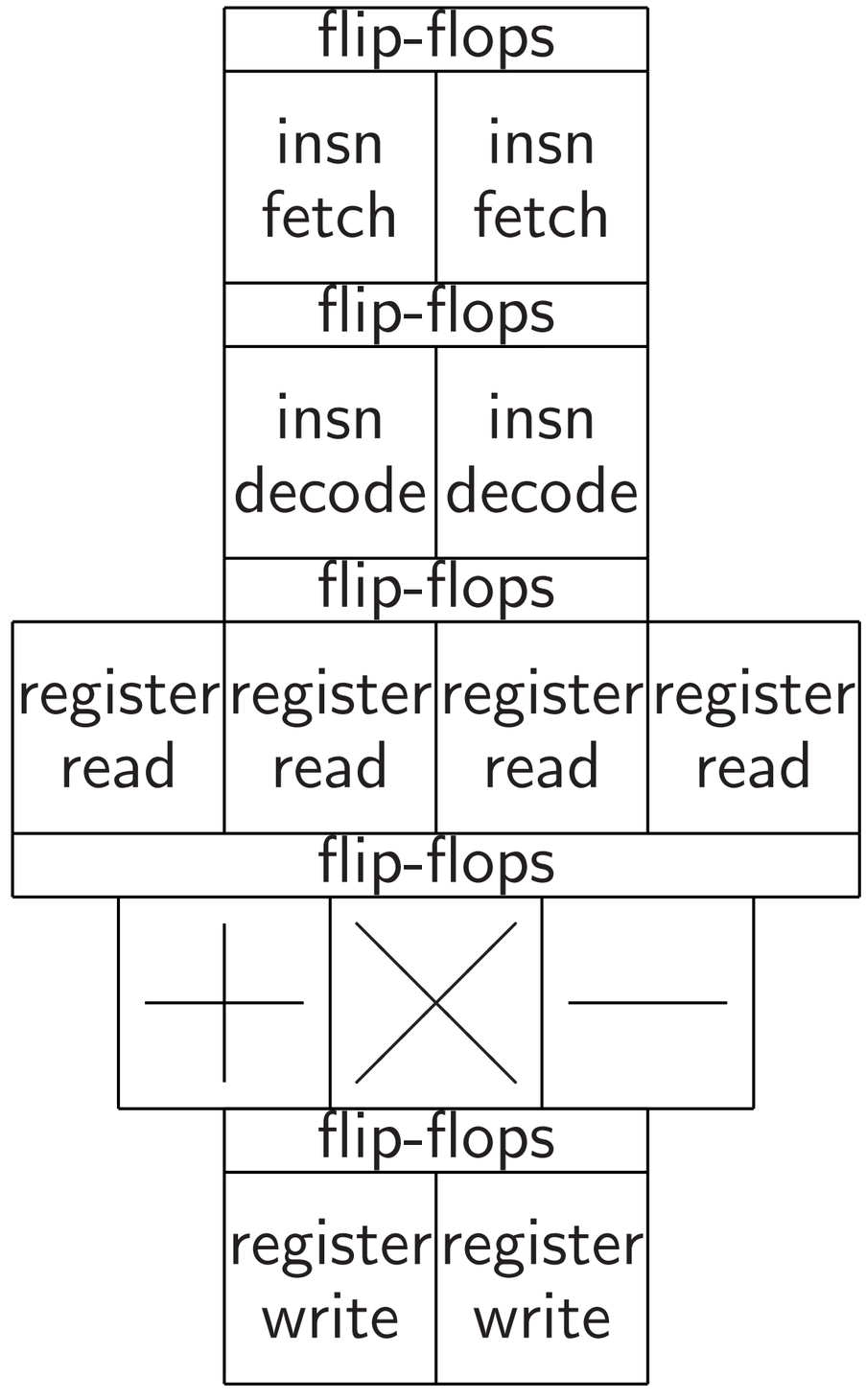

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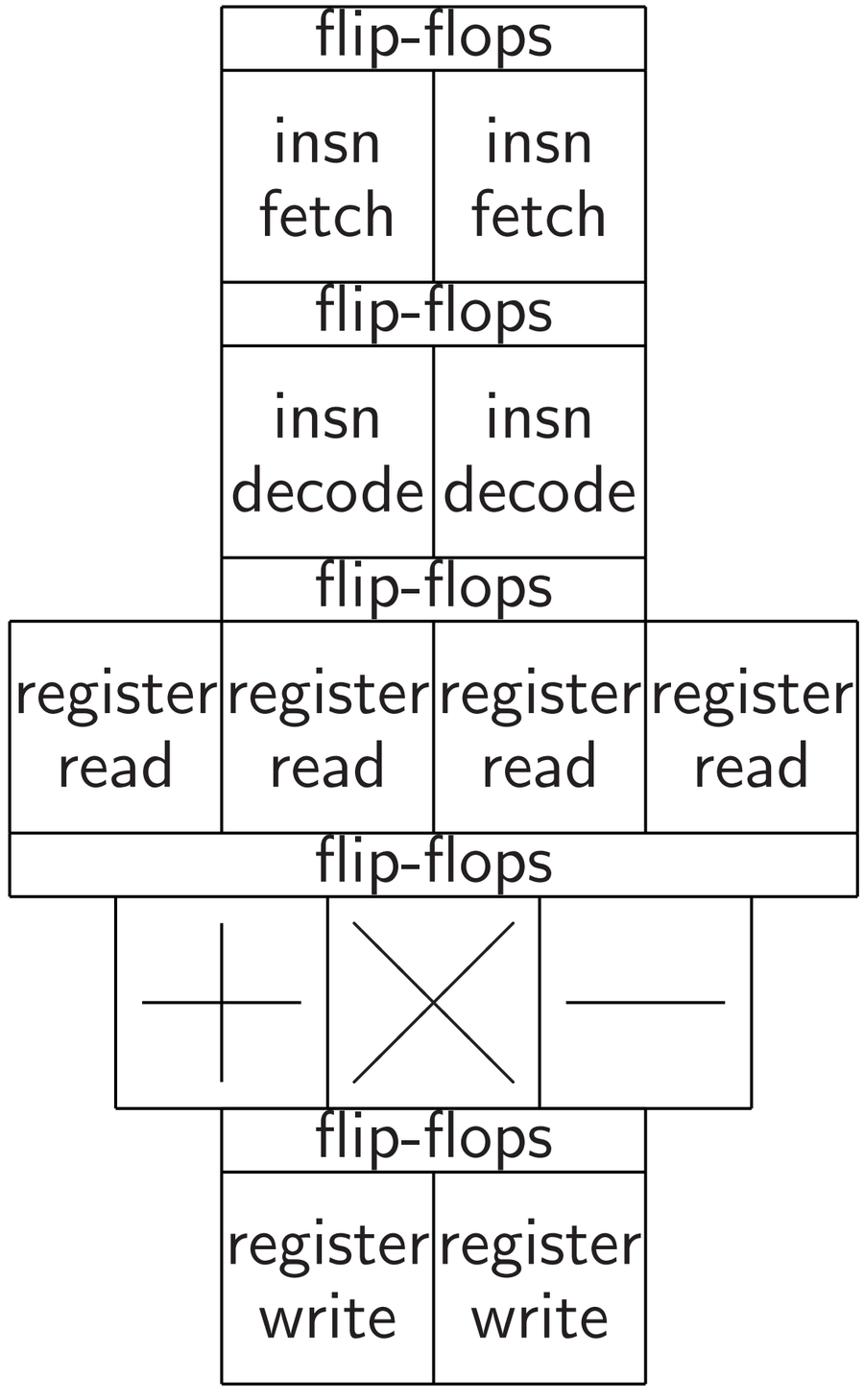

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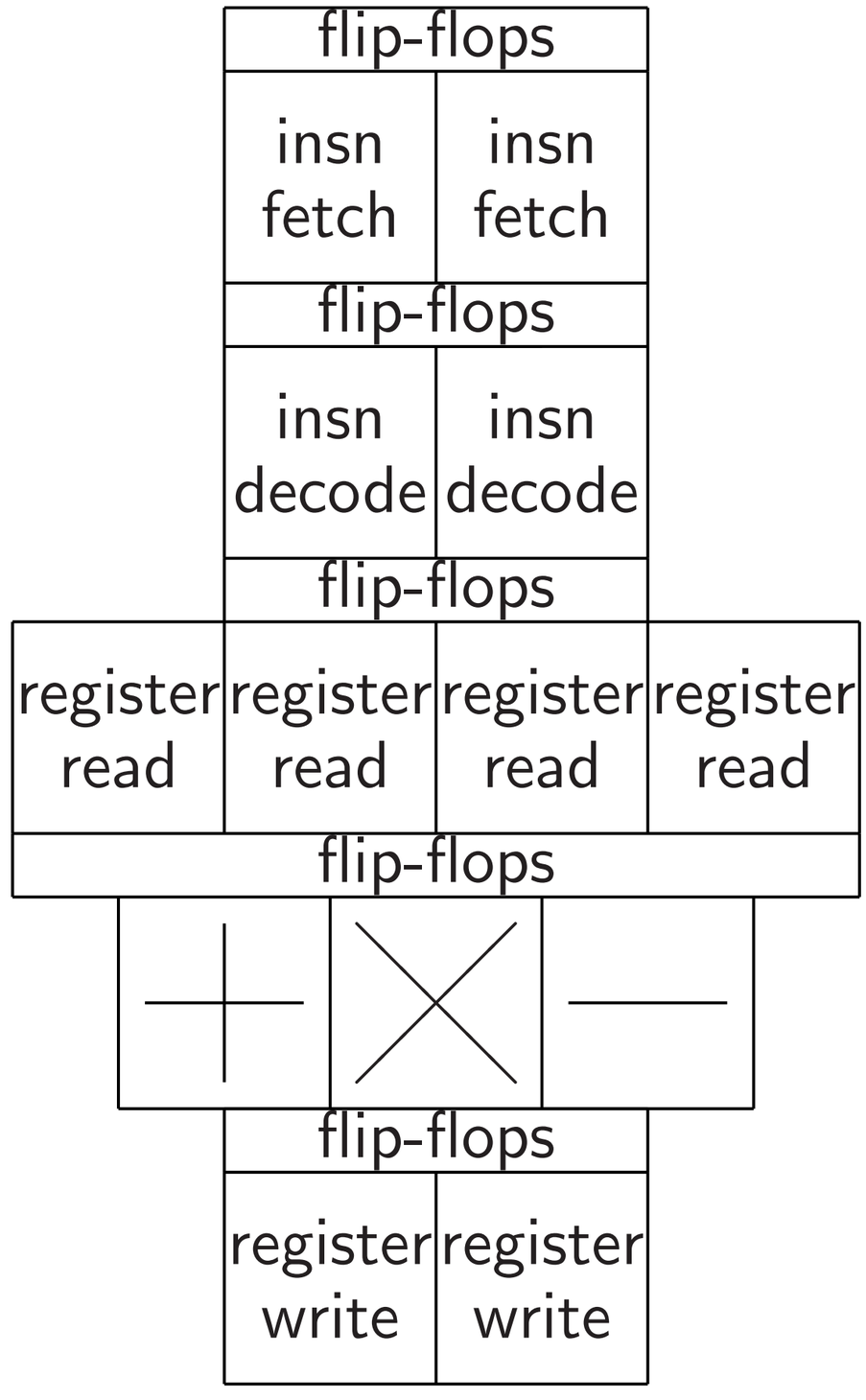

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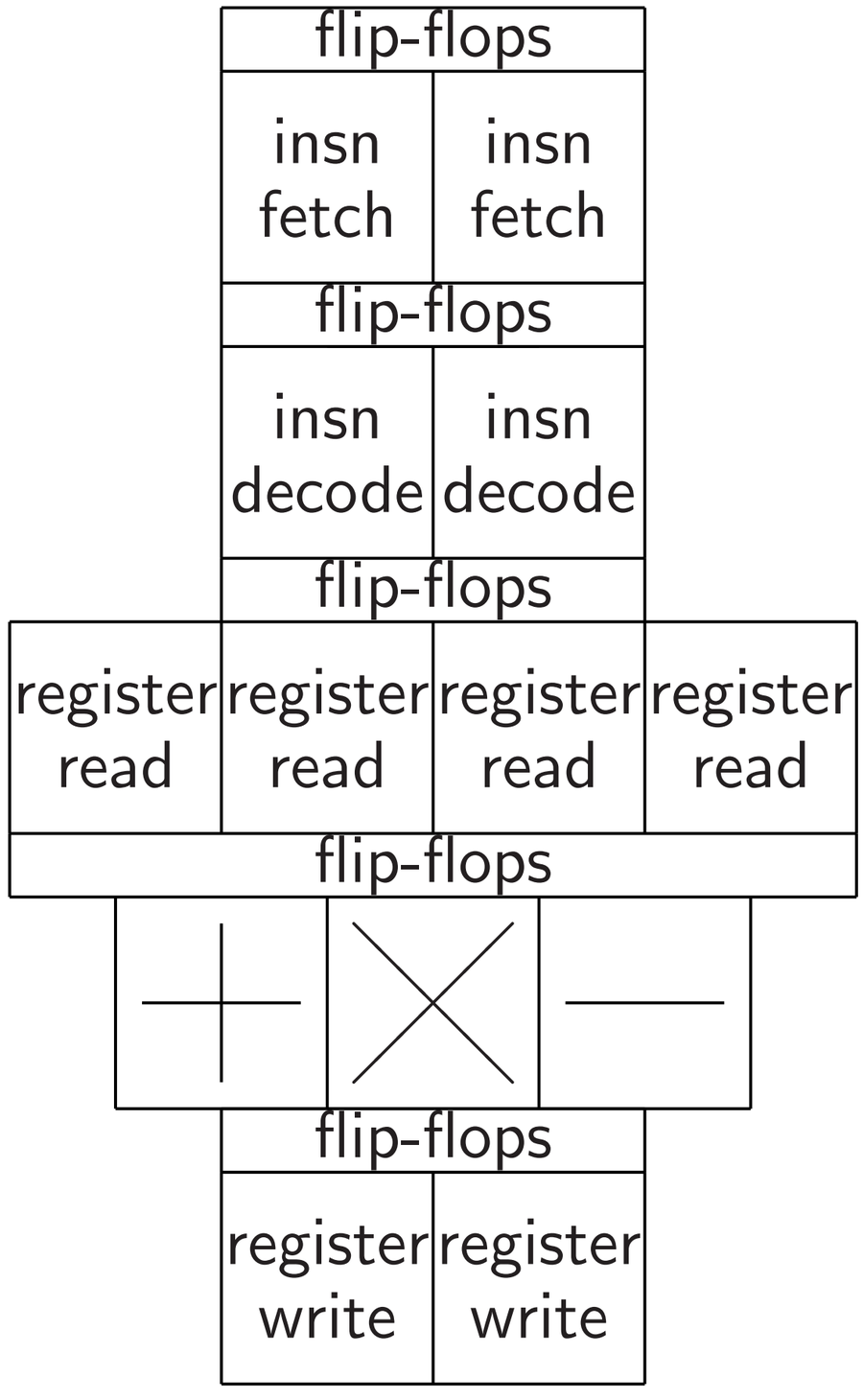

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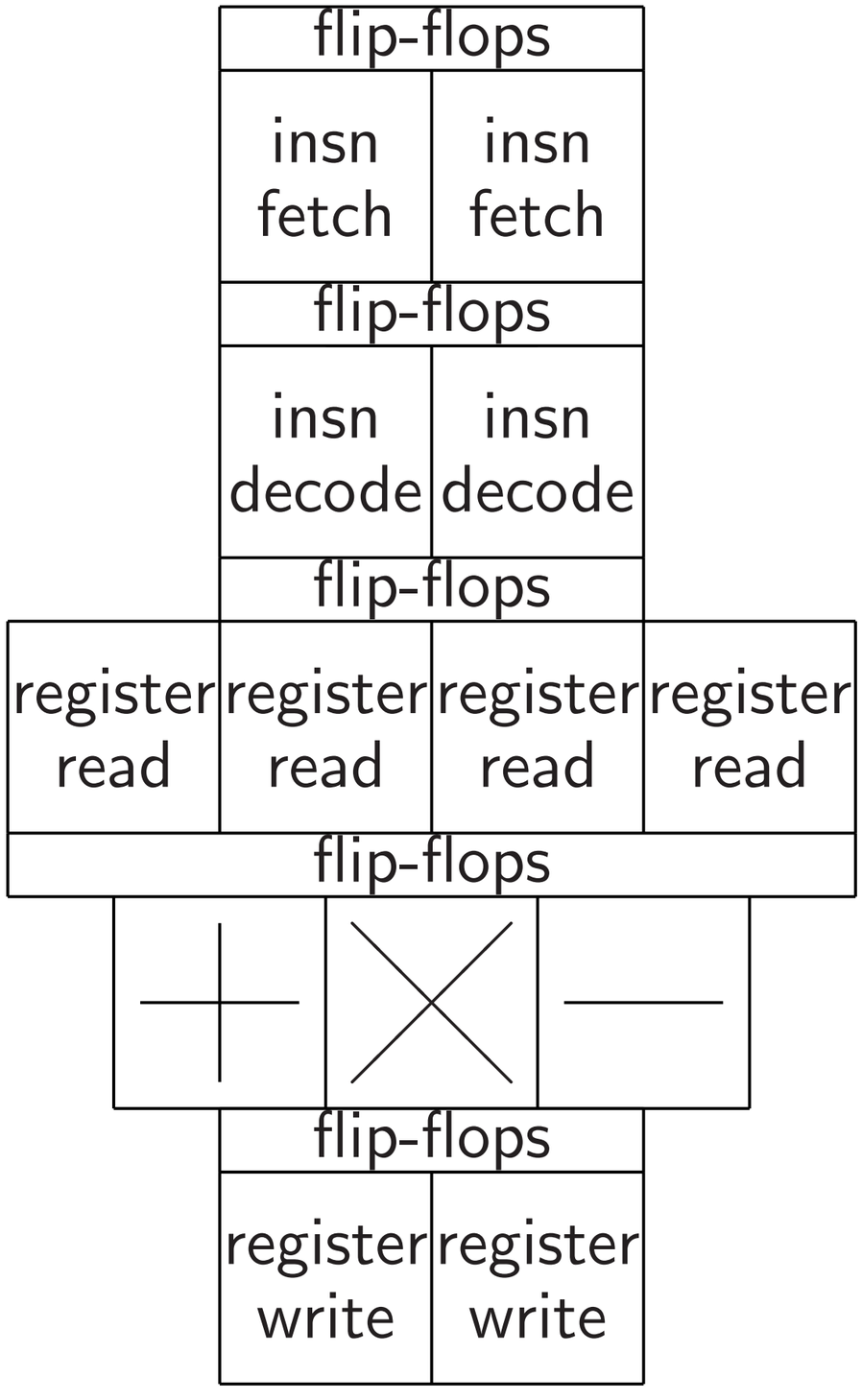

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Expand each 32-bit integer

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ARM "NEON" has $n = 4$;

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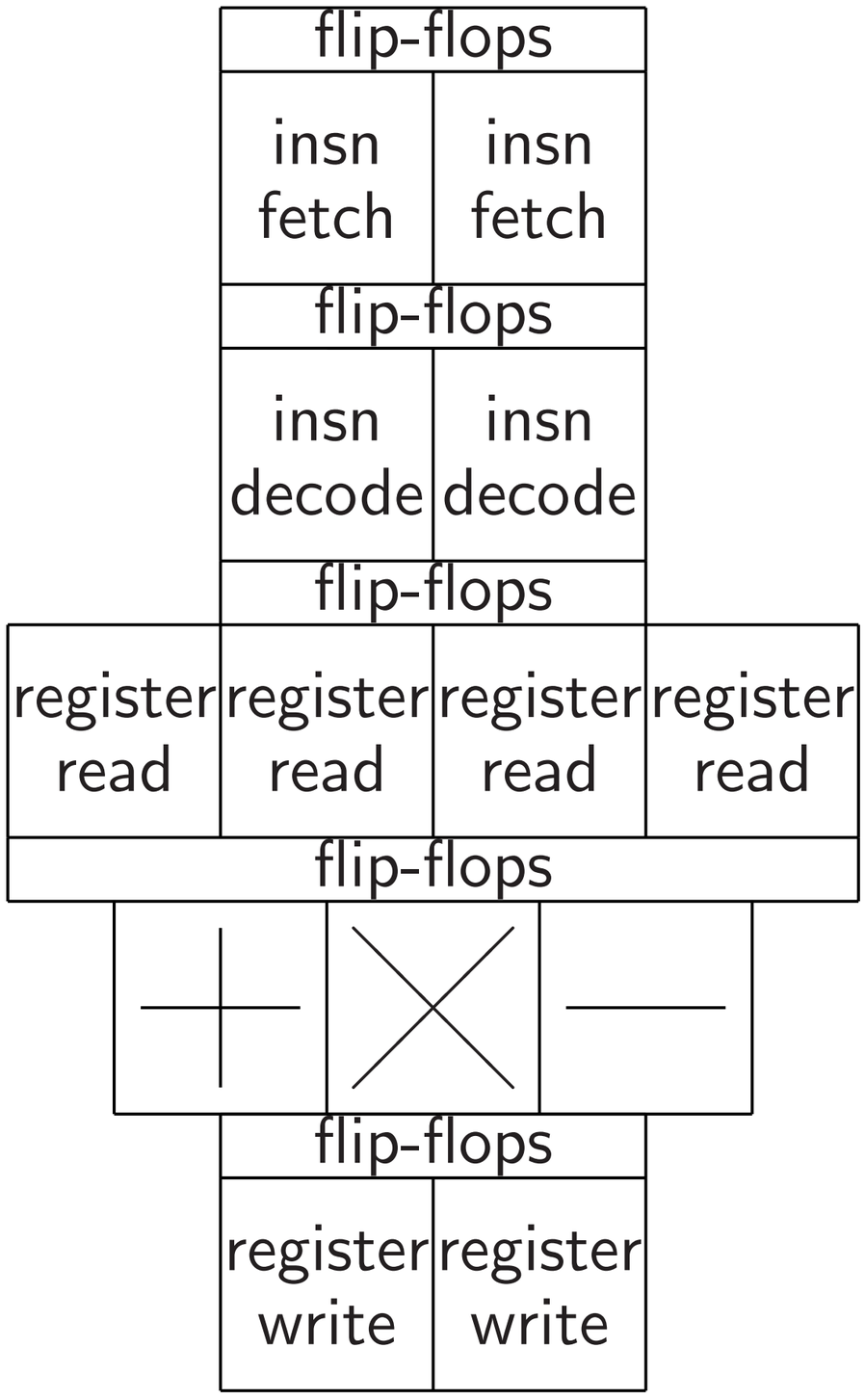

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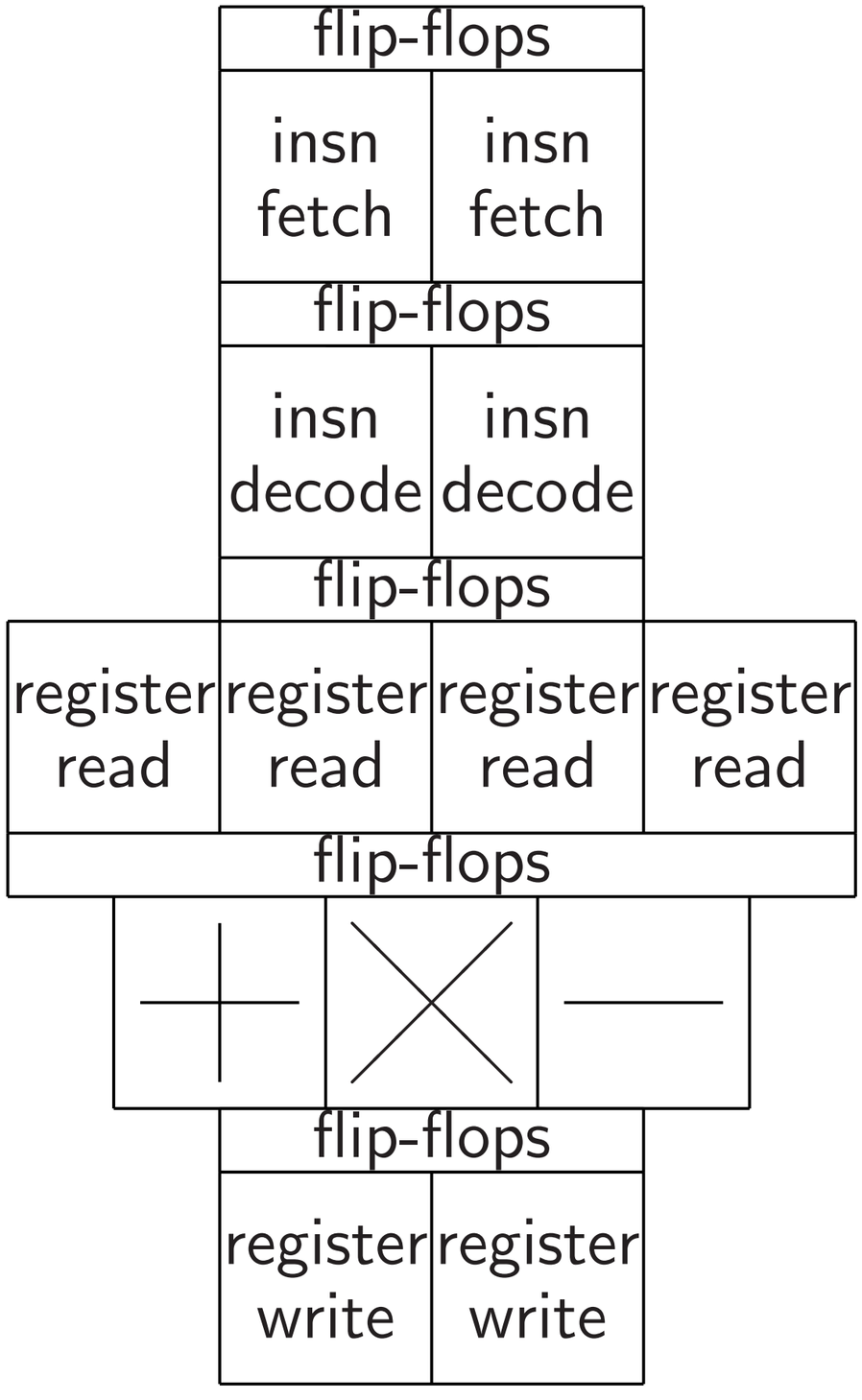

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Benefit: Amortizes insn circuits.

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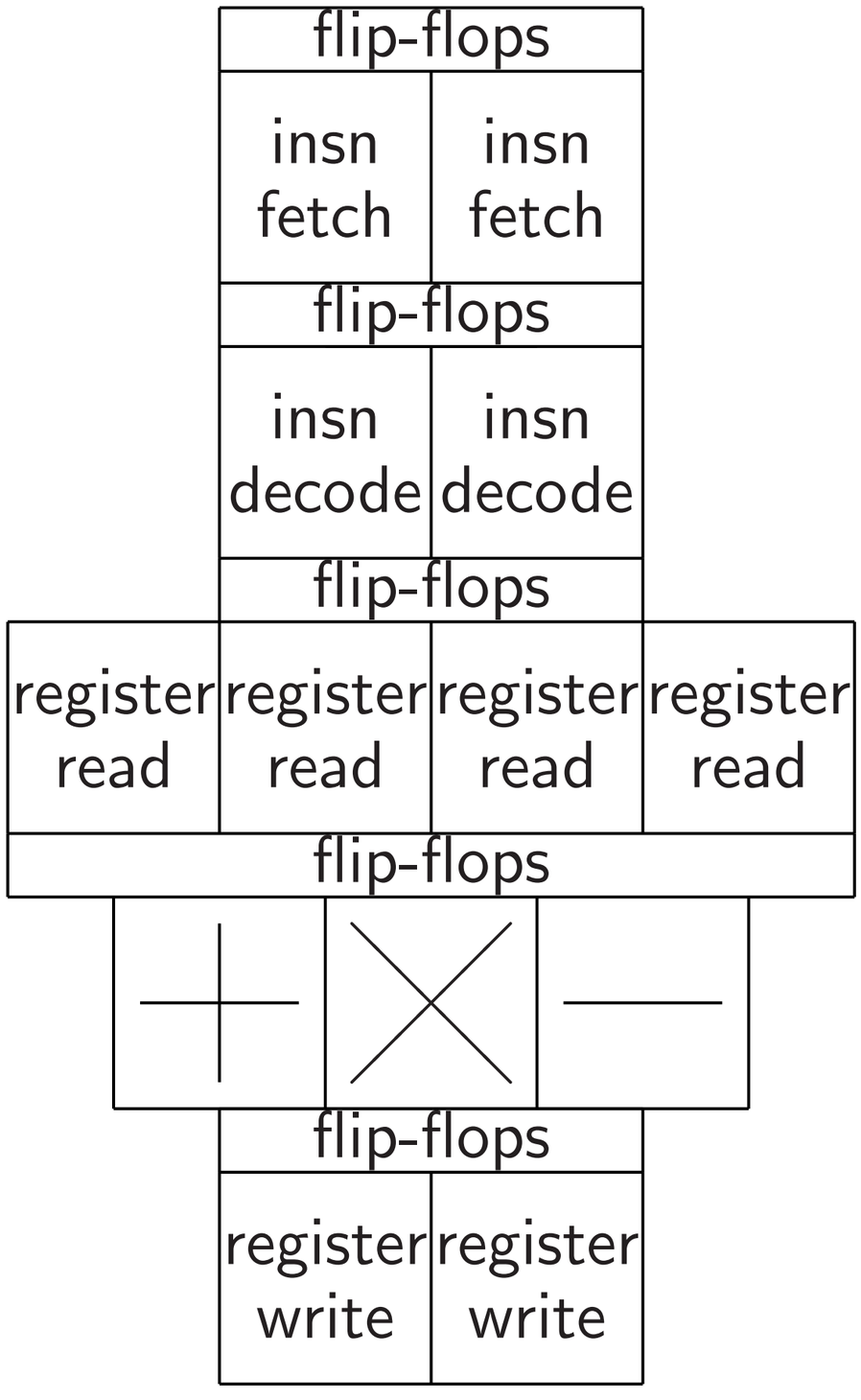

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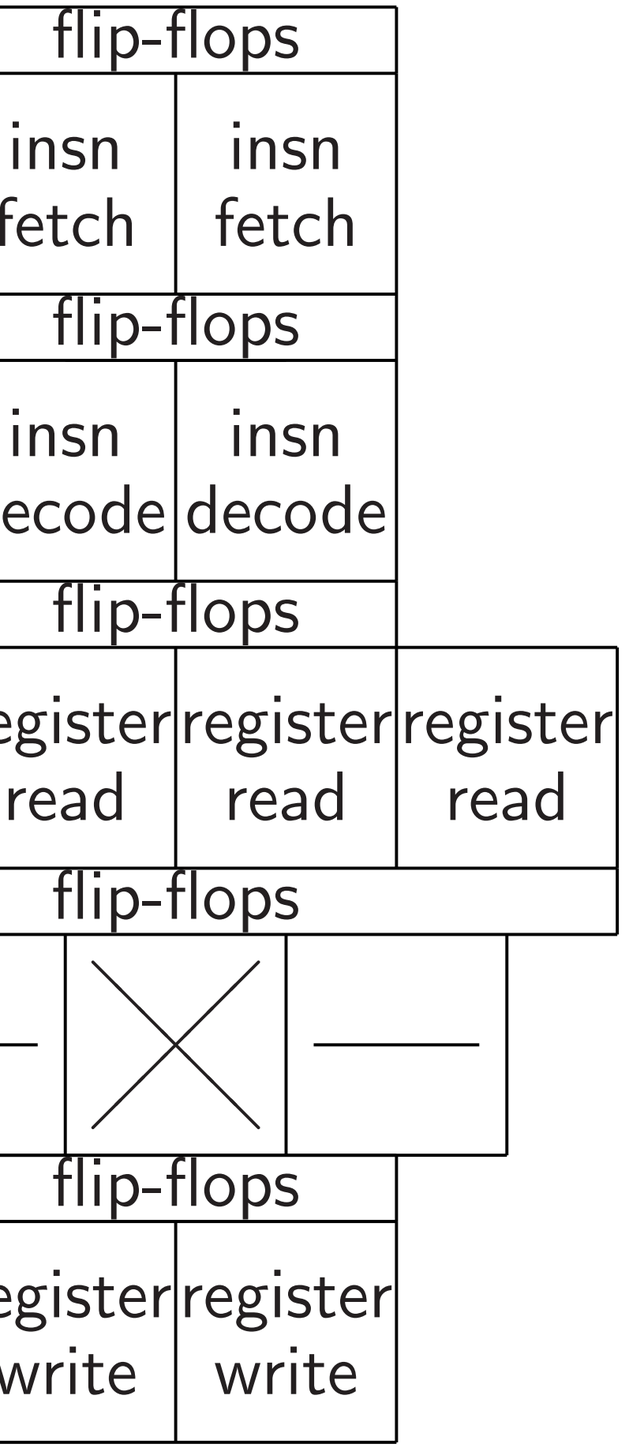

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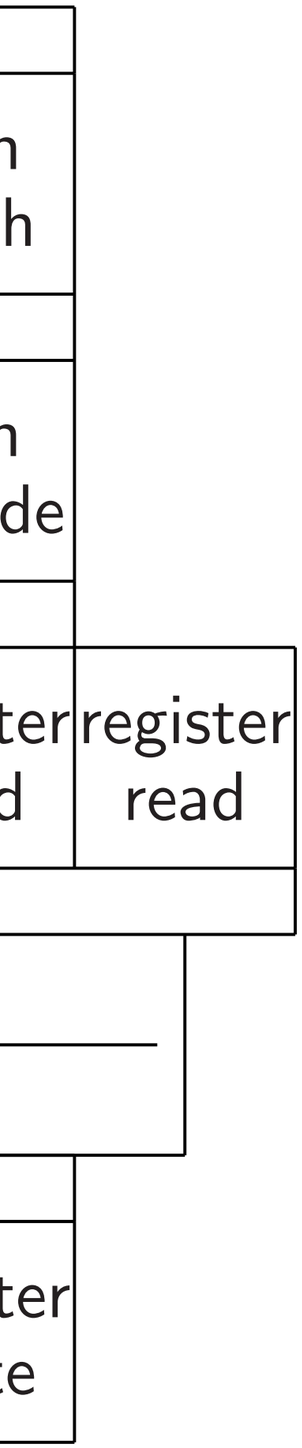

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Network on chip: the mesh

How expensive is sorting?

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Each number in $\{1, 2, \ldots, n$
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For simplicity assume $n = 4^k$.

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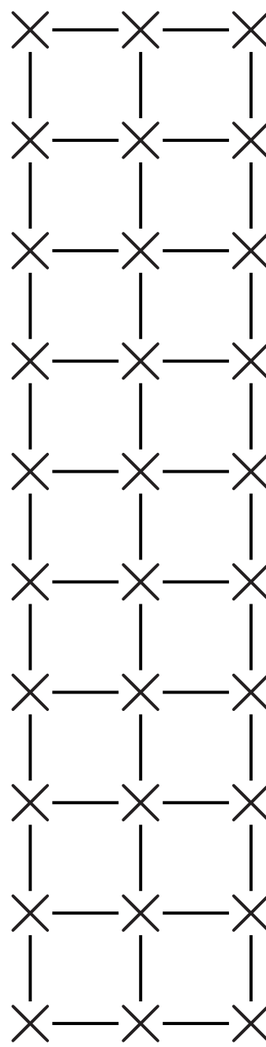

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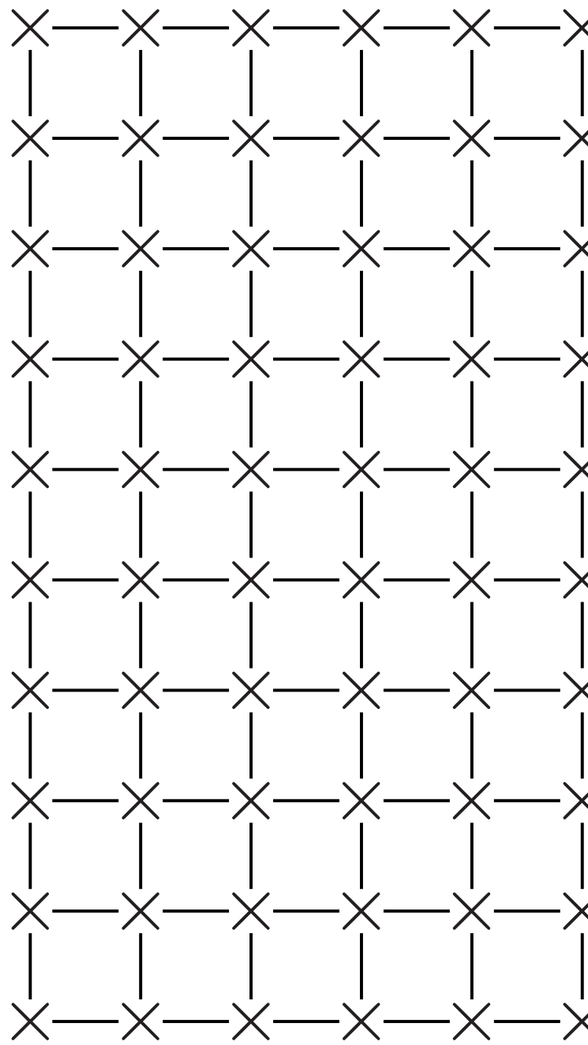

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Spread array across
square mesh of n small cells
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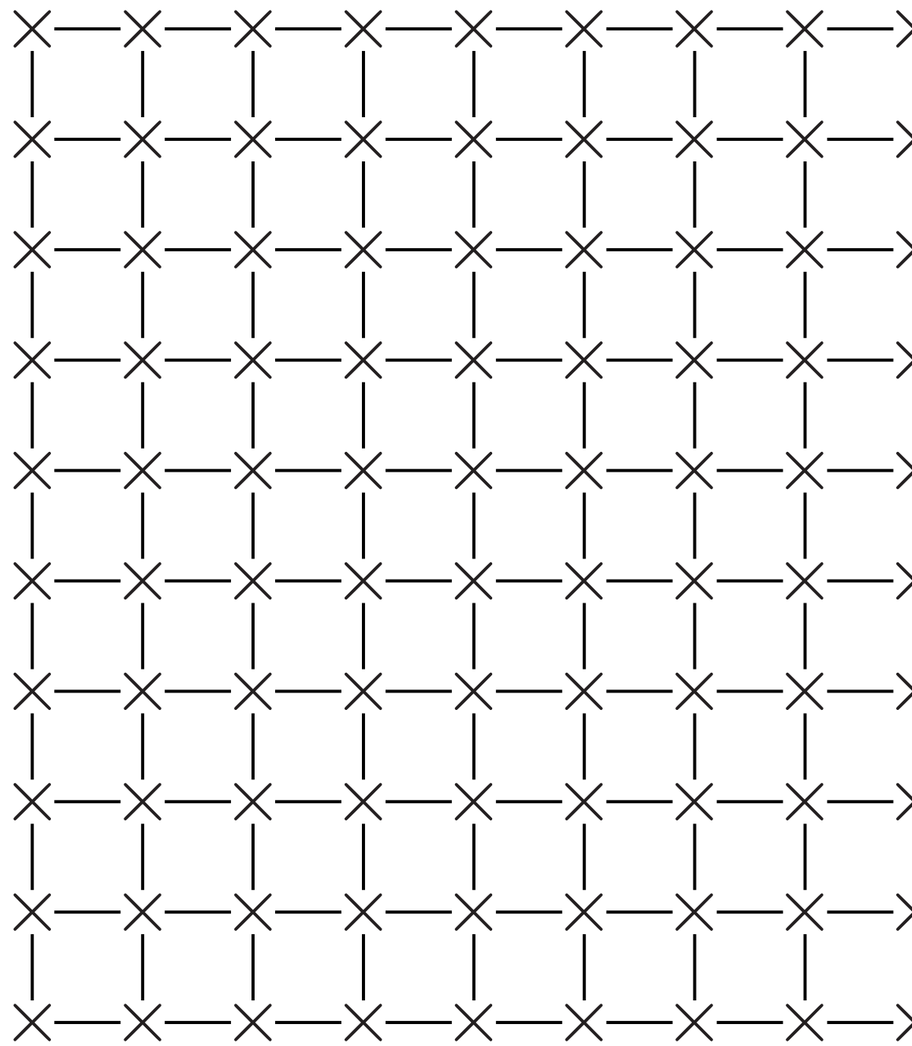

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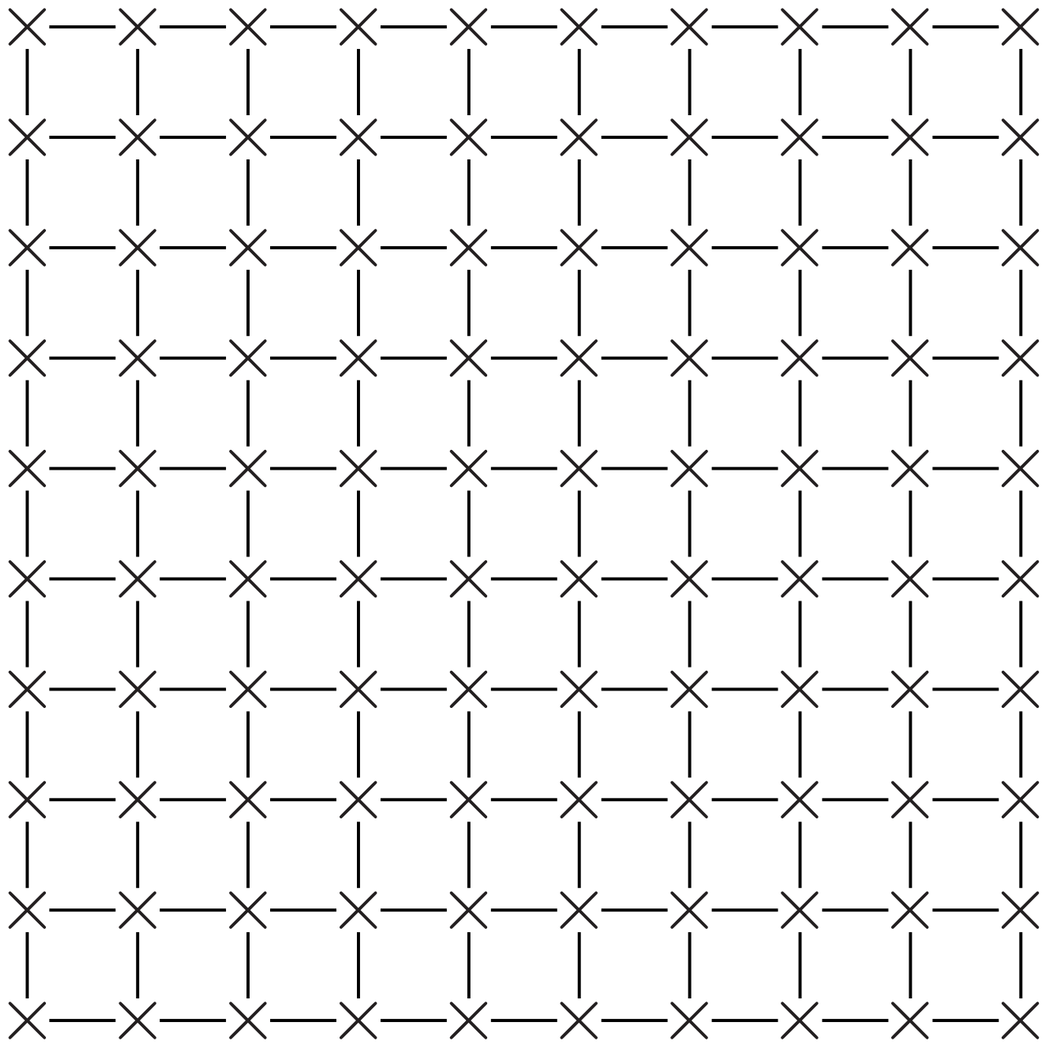

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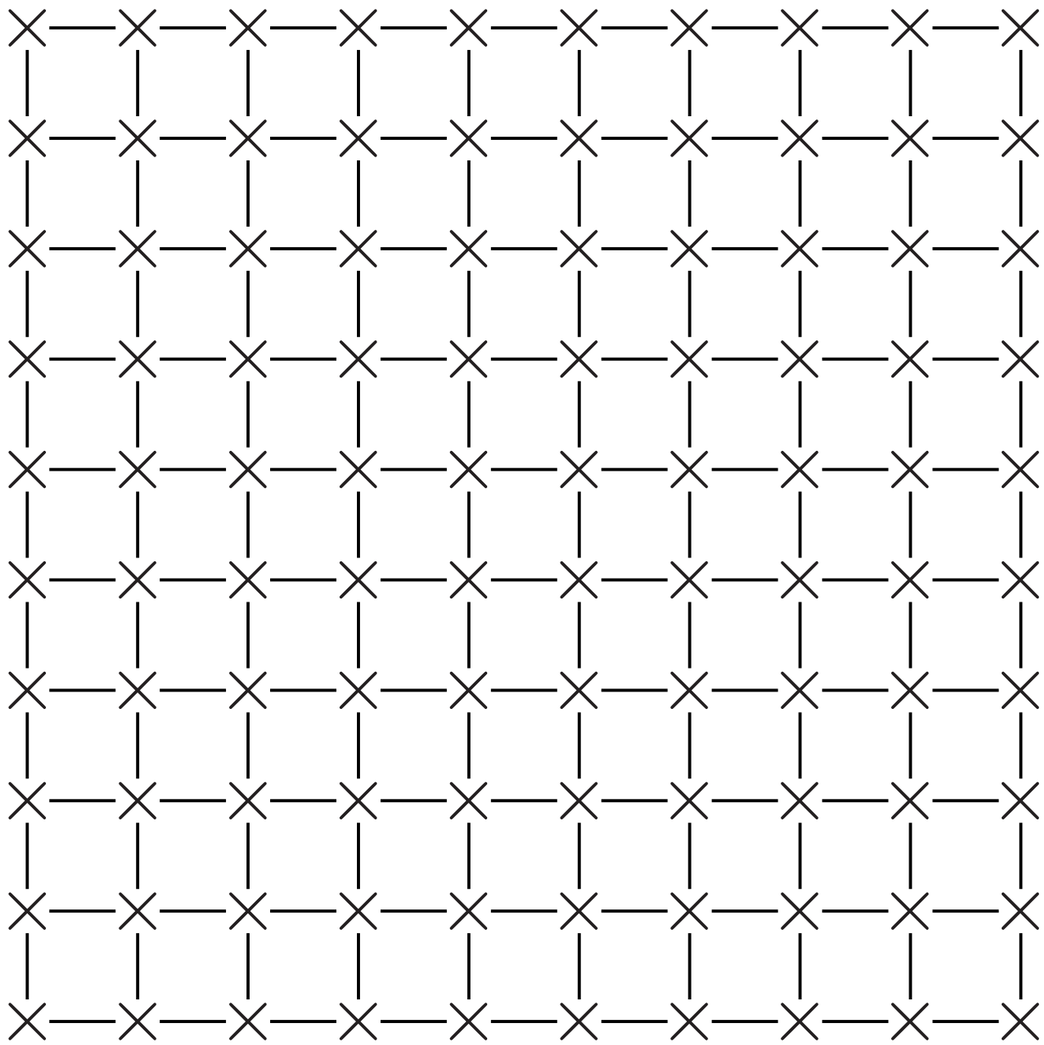
seconds used by
f area $n^{1+o(1)}$.

plicity assume $n = 4^k$.

Spread array across
square mesh of n small cells,
each of area $n^{o(1)}$,
with near-neighbor wiring:



Sort row
in $n^{0.5+}$

- Sort e
  3 1 4
  1 3 1
- Sort a
  1 3 1
  1 1 3
- Repea
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sorting?

numbers.
$1, 2, \ldots, n^2\}$,
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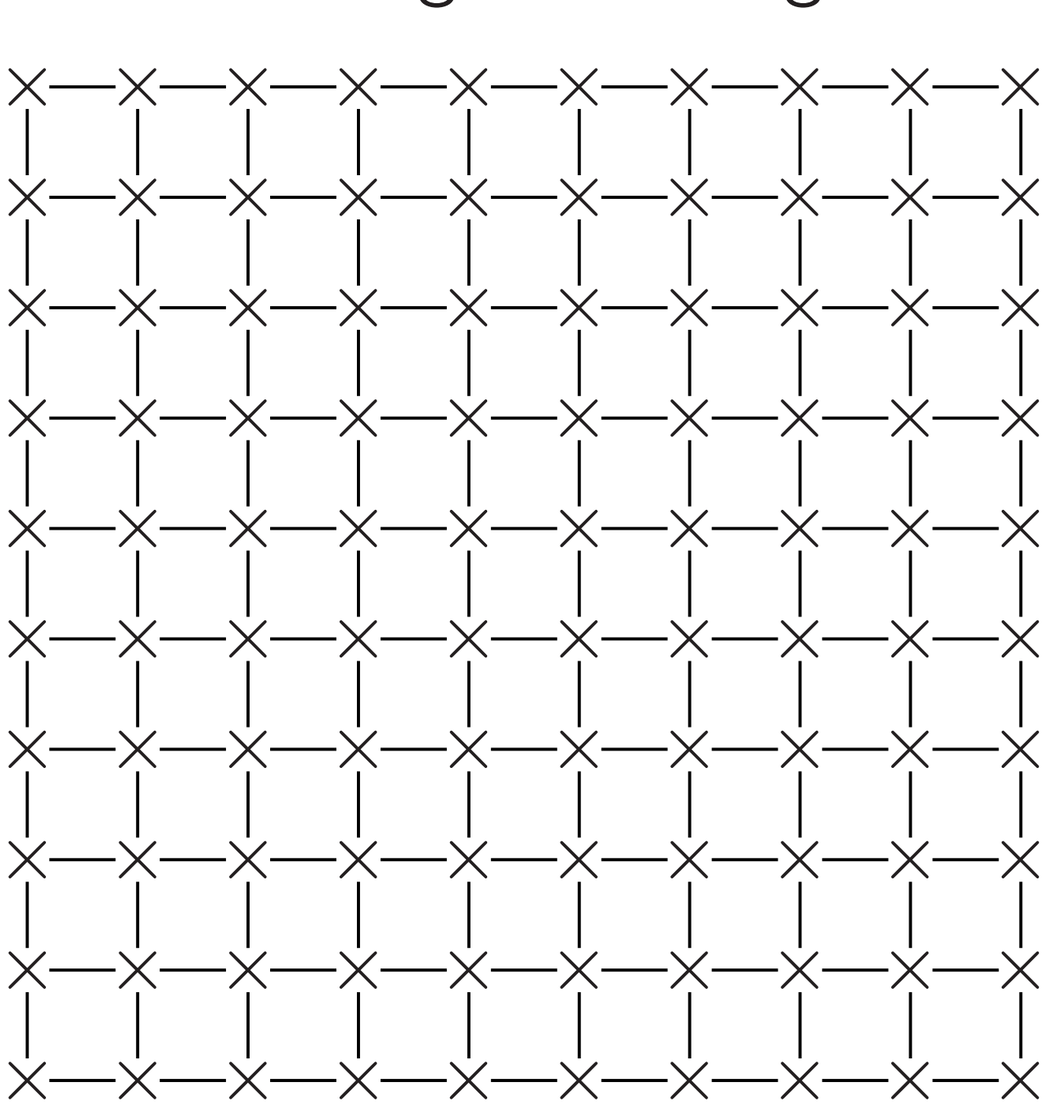
me $n = 4^k$.

Spread array across
square mesh of n small cells,
each of area $n^{o(1)}$,
with near-neighbor wiring:



Sort row of $n^{0.5}$ ce
in $n^{0.5+o(1)}$ second

- Sort each pair in
  3 1 4 1 5 9 2 6
  1 3 1 4 5 9 2 6
- Sort alternate pa
  1 3 1 4 5 9 2 6
  1 1 3 4 5 2 9 6
- Repeat until num
  equals row lengt

2},

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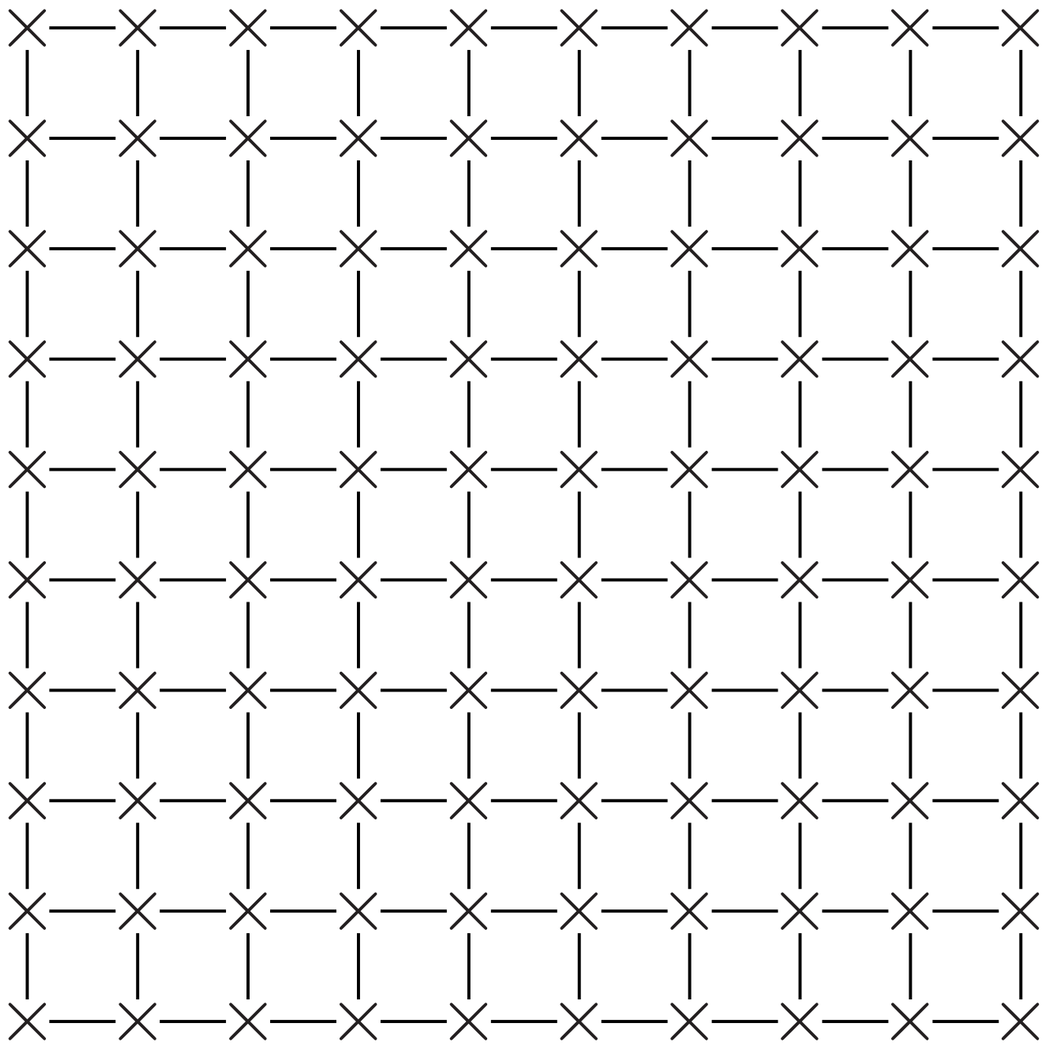
33

Spread array across
square mesh of n small cells,
each of area $n^{o(1)}$,
with near-neighbor wiring:

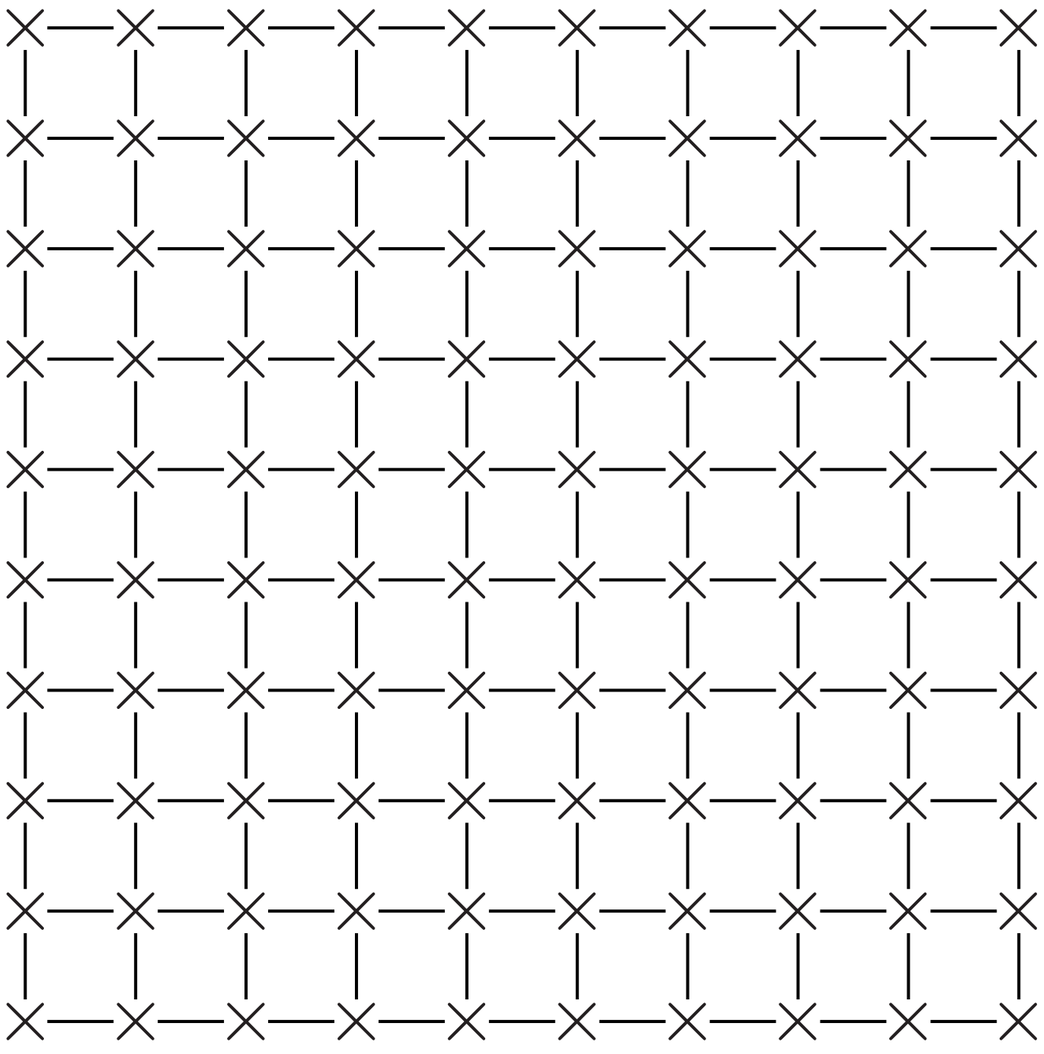


34

Sort row of $n^{0.5}$ cells
in $n^{0.5+o(1)}$ seconds:

- Sort each pair in parallel.
  $\underline{3\ 1}\ \underline{4\ 1}\ \underline{5\ 9}\ \underline{2\ 6} \mapsto$
  $1\ 3\ 1\ 4\ 5\ 9\ 2\ 6$
- Sort alternate pairs in para
  $1\ \underline{3\ 1}\ \underline{4\ 5}\ \underline{9\ 2}\ 6 \mapsto$
  $1\ 1\ 3\ 4\ 5\ 2\ 9\ 6$
- Repeat until number of st
  equals row length.

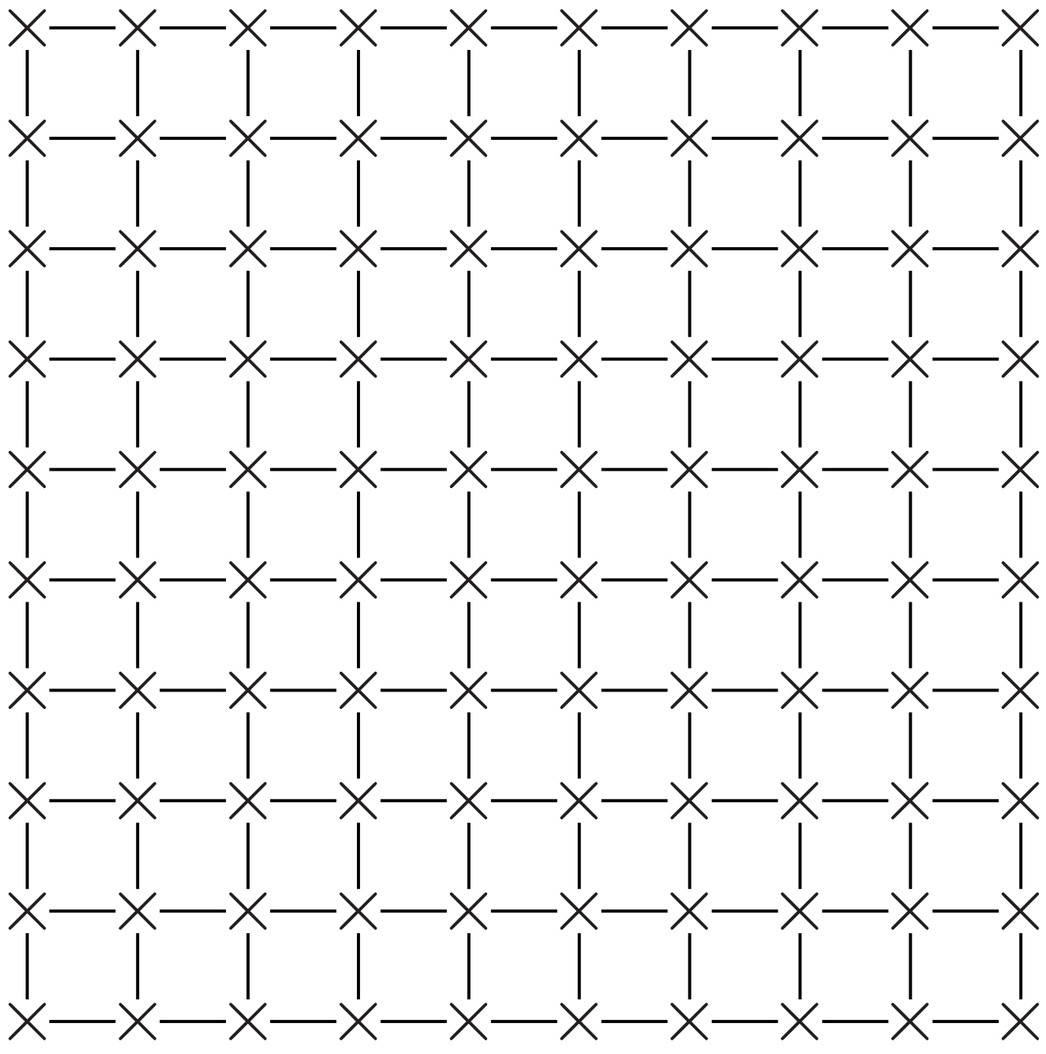
Spread array across
square mesh of n small cells,
each of area $n^{o(1)}$,
with near-neighbor wiring:



Sort row of $n^{0.5}$ cells
in $n^{0.5+o(1)}$ seconds:

- Sort each pair in parallel.
  3 1 4 1 5 9 2 6 $\mapsto$
  1 3 1 4 5 9 2 6
- Sort alternate pairs in parallel.
  1 3 1 4 5 9 2 6 $\mapsto$
  1 1 3 4 5 2 9 6
- Repeat until number of steps
  equals row length.

Spread array across
square mesh of n small cells,
each of area $n^{o(1)}$,
with near-neighbor wiring:

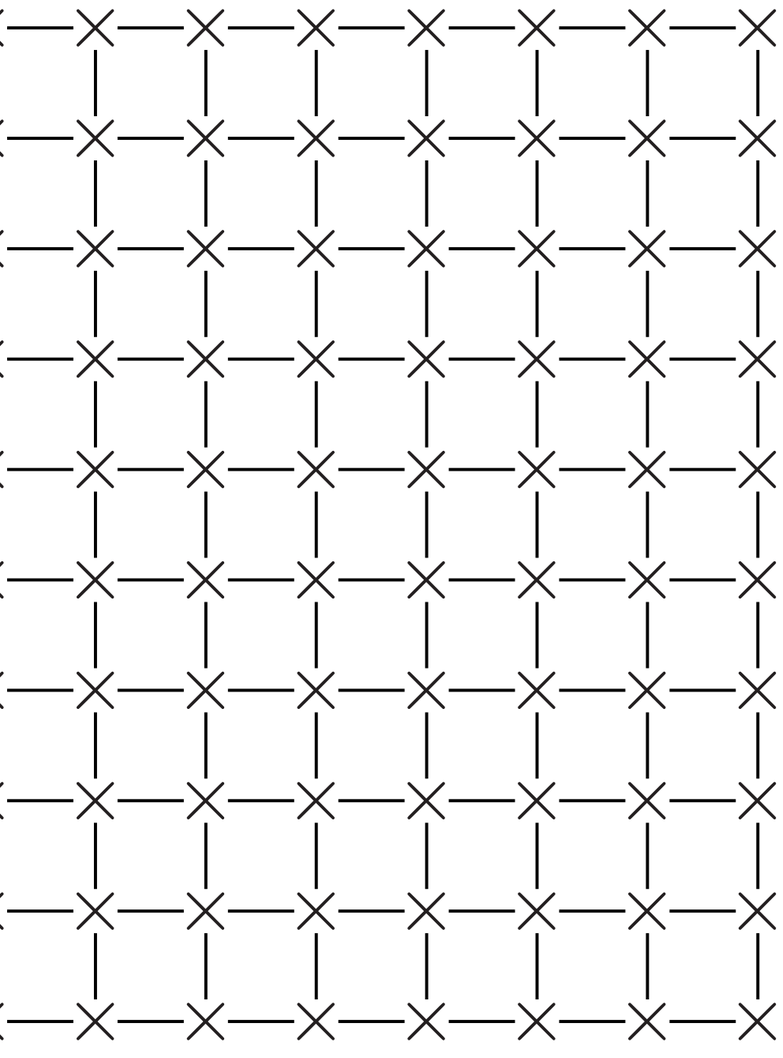


Sort row of $n^{0.5}$ cells
in $n^{0.5+o(1)}$ seconds:

- Sort each pair in parallel.
  $\underline{3\ 1}\ \underline{4\ 1}\ \underline{5\ 9}\ \underline{2\ 6} \mapsto$
  $1\ 3\ 1\ 4\ 5\ 9\ 2\ 6$
- Sort alternate pairs in parallel.
  $1\ \underline{3\ 1}\ \underline{4\ 5}\ \underline{9\ 2}\ 6 \mapsto$
  $1\ 1\ 3\ 4\ 5\ 2\ 9\ 6$
- Repeat until number of steps
  equals row length.

Sort *each* row, in parallel,
in a *total* of $n^{0.5+o(1)}$ seconds.

array across
nesh of n small cells,
area $n^{o(1)}$,
ar-neighbor wiring:



Sort row of $n^{0.5}$ cells
in $n^{0.5+o(1)}$ seconds:

- Sort each pair in parallel.
  $\underline{3\ 1}\ \underline{4\ 1}\ \underline{5\ 9}\ \underline{2\ 6} \mapsto$
  $1\ 3\ 1\ 4\ 5\ 9\ 2\ 6$
- Sort alternate pairs in parallel.
  $1\ \underline{3\ 1}\ \underline{4\ 5}\ \underline{9\ 2}\ 6 \mapsto$
  $1\ 1\ 3\ 4\ 5\ 2\ 9\ 6$
- Repeat until number of steps
  equals row length.

Sort *each* row, in parallel,
in a *total* of $n^{0.5+o(1)}$ seconds.

Sort all
in $n^{0.5+$

- Recurs
  in para
- Sort e
- Sort e
- Sort e
- Sort e

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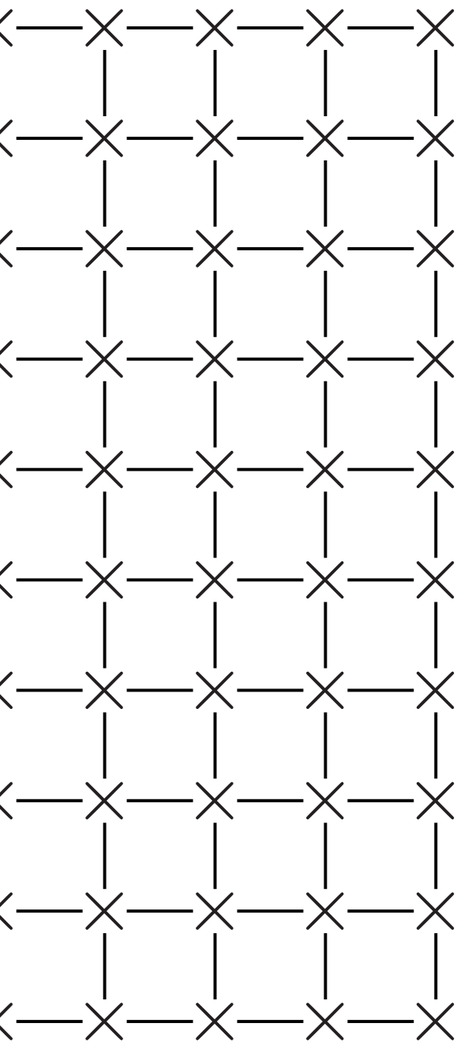

Sort row of $n^{0.5}$ cells
in $n^{0.5+o(1)}$ seconds:

- Sort each pair in parallel.
  $\underline{3\ 1}\ \underline{4\ 1}\ \underline{5\ 9}\ \underline{2\ 6} \mapsto$
  $1\ 3\ 1\ 4\ 5\ 9\ 2\ 6$
- Sort alternate pairs in parallel.
  $1\ \underline{3\ 1}\ \underline{4\ 5}\ \underline{9\ 2}\ 6 \mapsto$
  $1\ 1\ 3\ 4\ 5\ 2\ 9\ 6$
- Repeat until number of steps
  equals row length.

Sort *each* row, in parallel,
in a *total* of $n^{0.5+o(1)}$ seconds.

Sort all n cells
in $n^{0.5+o(1)}$ second

- Recursively sort
  in parallel, if $n >$
- Sort each colum
- Sort each row in
- Sort each colum
- Sort each row in

With proper choic
left-to-right/right-
for each row, can
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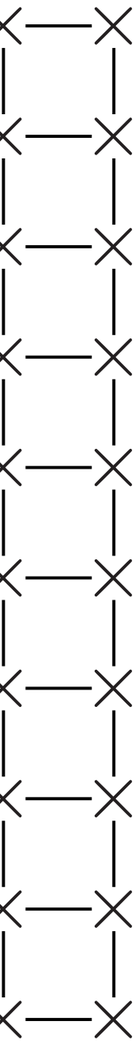

Sort row of $n^{0.5}$ cells
in $n^{0.5+o(1)}$ seconds:

- Sort each pair in parallel.
  $\underline{3\ 1}\ \underline{4\ 1}\ \underline{5\ 9}\ \underline{2\ 6} \mapsto$
  $1\ 3\ 1\ 4\ 5\ 9\ 2\ 6$
- Sort alternate pairs in parallel.
  $1\ \underline{3\ 1}\ \underline{4\ 5}\ \underline{9\ 2}\ 6 \mapsto$
  $1\ 1\ 3\ 4\ 5\ 2\ 9\ 6$
- Repeat until number of steps
  equals row length.

Sort *each* row, in parallel,
in a *total* of $n^{0.5+o(1)}$ seconds.

Sort all n cells
in $n^{0.5+o(1)}$ seconds:

- Recursively sort quadrants
  in parallel, if $n > 1$.
- Sort each column in parall
- Sort each row in parallel.
- Sort each column in parall
- Sort each row in parallel.

With proper choice of
left-to-right/right-to-left
for each row, can prove
that this sorts whole array.

Sort row of $n^{0.5}$ cells
in $n^{0.5+o(1)}$ seconds:

- Sort each pair in parallel.
  $\underline{3\ 1}\ \underline{4\ 1}\ \underline{5\ 9}\ \underline{2\ 6} \mapsto$
  $1\ 3\ 1\ 4\ 5\ 9\ 2\ 6$
- Sort alternate pairs in parallel.
  $1\ \underline{3\ 1}\ \underline{4\ 5}\ \underline{9\ 2}\ 6 \mapsto$
  $1\ 1\ 3\ 4\ 5\ 2\ 9\ 6$
- Repeat until number of steps
  equals row length.

Sort *each* row, in parallel,
in a *total* of $n^{0.5+o(1)}$ seconds.

Sort all n cells
in $n^{0.5+o(1)}$ seconds:

- Recursively sort quadrants
  in parallel, if $n > 1$.
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- Sort each row in parallel.
- Sort each column in parallel.
- Sort each row in parallel.

With proper choice of
left-to-right/right-to-left
for each row, can prove
that this sorts whole array.

of $n^{0.5}$ cells
$^{o(1)}$ seconds:

ach pair in parallel.

1 5 9 2 6 $\mapsto$

4 5 9 2 6

lternate pairs in parallel.

4 5 9 2 6 $\mapsto$

4 5 2 9 6

t until number of steps
row length.

h row, in parallel,
al of $n^{0.5+o(1)}$ seconds.

Sort all n cells
in $n^{0.5+o(1)}$ seconds:

- Recursively sort quadrants
  in parallel, if $n > 1$.
- Sort each column in parallel.
- Sort each row in parallel.
- Sort each column in parallel.
- Sort each row in parallel.

With proper choice of
left-to-right/right-to-left
for each row, can prove
that this sorts whole array.

For exam
this $8 \times$

3	1	4
5	3	5
2	3	8
3	3	8
0	2	8
1	6	9
5	1	0
7	4	9

ells

ds:

n parallel.

$\mapsto$

airs in parallel.

$\mapsto$

mber of steps

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parallel,

$^{o(1)}$ seconds.

Sort all n cells
in $n^{0.5+o(1)}$ seconds:

- Recursively sort quadrants
  in parallel, if $n > 1$.
- Sort each column in parallel.
- Sort each row in parallel.
- Sort each column in parallel.
- Sort each row in parallel.

With proper choice of
left-to-right/right-to-left
for each row, can prove
that this sorts whole array.

For example, assu

this 8×8 array is

3	1	4	1	5	9
5	3	5	8	9	7
2	3	8	4	6	2
3	3	8	3	2	7
0	2	8	8	4	1
1	6	9	3	9	9
5	1	0	5	8	2
7	4	9	4	4	5

allel.

eps

ds.

Sort all n cells
in $n^{0.5+o(1)}$ seconds:

- Recursively sort quadrants
  in parallel, if $n > 1$.
- Sort each column in parallel.
- Sort each row in parallel.
- Sort each column in parallel.
- Sort each row in parallel.

With proper choice of
left-to-right/right-to-left
for each row, can prove
that this sorts whole array.

For example, assume that
this 8×8 array is in cells:

3	1	4	1	5	9	2	6
5	3	5	8	9	7	9	3
2	3	8	4	6	2	6	4
3	3	8	3	2	7	9	5
0	2	8	8	4	1	9	7
1	6	9	3	9	9	3	7
5	1	0	5	8	2	0	9
7	4	9	4	4	5	9	2

Sort all n cells
in $n^{0.5+o(1)}$ seconds:

- Recursively sort quadrants
  in parallel, if $n > 1$.
- Sort each column in parallel.
- Sort each row in parallel.
- Sort each column in parallel.
- Sort each row in parallel.

With proper choice of
left-to-right/right-to-left
for each row, can prove
that this sorts whole array.

For example, assume that
this 8×8 array is in cells:

3	1	4	1	5	9	2	6
5	3	5	8	9	7	9	3
2	3	8	4	6	2	6	4
3	3	8	3	2	7	9	5
0	2	8	8	4	1	9	7
1	6	9	3	9	9	3	7
5	1	0	5	8	2	0	9
7	4	9	4	4	5	9	2

n cells
$^{(1)}$ seconds:

sively sort quadrants
allel, if $n > 1$.
ach column in parallel.
ach row in parallel.
ach column in parallel.
ach row in parallel.

oper choice of
ght/right-to-left
row, can prove
s sorts whole array.

For example, assume that
this 8×8 array is in cells:

3	1	4	1	5	9	2	6
5	3	5	8	9	7	9	3
2	3	8	4	6	2	6	4
3	3	8	3	2	7	9	5
0	2	8	8	4	1	9	7
1	6	9	3	9	9	3	7
5	1	0	5	8	2	0	9
7	4	9	4	4	5	9	2

Recursiv
top $\rightarrow$,

1	1	2
3	3	3
3	4	4
5	8	8
1	1	0
4	4	3
7	6	5
9	9	8

ds:

quadrants
> 1.
n in parallel.
parallel.
n in parallel.
parallel.

e of
to-left
prove
ble array.

For example, assume that this 8×8 array is in cells:

3	1	4	1	5	9	2	6
5	3	5	8	9	7	9	3
2	3	8	4	6	2	6	4
3	3	8	3	2	7	9	5
0	2	8	8	4	1	9	7
1	6	9	3	9	9	3	7
5	1	0	5	8	2	0	9
7	4	9	4	4	5	9	2

Recursively sort qu

top $\rightarrow$, bottom $\leftarrow$

1	1	2	3	2	2
3	3	3	3	4	5
3	4	4	5	6	6
5	8	8	8	9	9
1	1	0	0	2	2
4	4	3	2	5	4
7	6	5	5	9	8
9	9	8	8	9	9

el.

el.

For example, assume that this 8×8 array is in cells:

3	1	4	1	5	9	2	6
5	3	5	8	9	7	9	3
2	3	8	4	6	2	6	4
3	3	8	3	2	7	9	5
0	2	8	8	4	1	9	7
1	6	9	3	9	9	3	7
5	1	0	5	8	2	0	9
7	4	9	4	4	5	9	2

Recursively sort quadrants, top $\rightarrow$, bottom $\leftarrow$:

1	1	2	3	2	2	2	3
3	3	3	3	4	5	5	6
3	4	4	5	6	6	7	7
5	8	8	8	9	9	9	9
1	1	0	0	2	2	1	0
4	4	3	2	5	4	4	3
7	6	5	5	9	8	7	7
9	9	8	8	9	9	9	9

For example, assume that this 8×8 array is in cells:

3	1	4	1	5	9	2	6
5	3	5	8	9	7	9	3
2	3	8	4	6	2	6	4
3	3	8	3	2	7	9	5
0	2	8	8	4	1	9	7
1	6	9	3	9	9	3	7
5	1	0	5	8	2	0	9
7	4	9	4	4	5	9	2

Recursively sort quadrants, top $\rightarrow$, bottom $\leftarrow$:

1	1	2	3	2	2	2	3
3	3	3	3	4	5	5	6
3	4	4	5	6	6	7	7
5	8	8	8	9	9	9	9
1	1	0	0	2	2	1	0
4	4	3	2	5	4	4	3
7	6	5	5	9	8	7	7
9	9	8	8	9	9	9	9

mple, assume that
8 array is in cells:

4	1	5	9	2	6
5	8	9	7	9	3
3	4	6	2	6	4
3	3	2	7	9	5
3	8	4	1	9	7
9	3	9	9	3	7
0	5	8	2	0	9
9	4	4	5	9	2

Recursively sort quadrants,
top $\rightarrow$, bottom $\leftarrow$:

1	1	2	3	2	2	2	3
3	3	3	3	4	5	5	6
3	4	4	5	6	6	7	7
5	8	8	8	9	9	9	9
1	1	0	0	2	2	1	0
4	4	3	2	5	4	4	3
7	6	5	5	9	8	7	7
9	9	8	8	9	9	9	9

Sort eac
in paralle

1	1	0
1	1	2
3	3	3
3	4	3
4	4	4
5	6	5
7	8	8
9	9	8

me that
in cells:

0	2	6
7	9	3
2	6	4
7	9	5
	9	7
0	3	7
2	0	9
5	9	2

Recursively sort quadrants,
top $\rightarrow$, bottom $\leftarrow$:

1	1	2	3	2	2	2	3
3	3	3	3	4	5	5	6
3	4	4	5	6	6	7	7
5	8	8	8	9	9	9	9
1	1	0	0	2	2	1	0
4	4	3	2	5	4	4	3
7	6	5	5	9	8	7	7
9	9	8	8	9	9	9	9

Sort each column
in parallel:

1	1	0	0	2	2
1	1	2	2	2	2
3	3	3	3	4	4
3	4	3	3	5	5
4	4	4	5	6	6
5	6	5	5	9	8
7	8	8	8	9	9
9	9	8	8	9	9

Recursively sort quadrants,
top $\rightarrow$, bottom $\leftarrow$:

1	1	2	3	2	2	2	3
3	3	3	3	4	5	5	6
3	4	4	5	6	6	7	7
5	8	8	8	9	9	9	9
1	1	0	0	2	2	1	0
4	4	3	2	5	4	4	3
7	6	5	5	9	8	7	7
9	9	8	8	9	9	9	9

Sort each column
in parallel:

1	1	0	0	2	2	1	0
1	1	2	2	2	2	2	3
3	3	3	3	4	4	4	3
3	4	3	3	5	5	5	6
4	4	4	5	6	6	7	7
5	6	5	5	9	8	7	7
7	8	8	8	9	9	9	9
9	9	8	8	9	9	9	9

Recursively sort quadrants,

top $\rightarrow$, bottom $\leftarrow$:

1	1	2	3	2	2	2	3
3	3	3	3	4	5	5	6
3	4	4	5	6	6	7	7
5	8	8	8	9	9	9	9
1	1	0	0	2	2	1	0
4	4	3	2	5	4	4	3
7	6	5	5	9	8	7	7
9	9	8	8	9	9	9	9

Sort each column

in parallel:

1	1	0	0	2	2	1	0
1	1	2	2	2	2	2	3
3	3	3	3	4	4	4	3
3	4	3	3	5	5	5	6
4	4	4	5	6	6	7	7
5	6	5	5	9	8	7	7
7	8	8	8	9	9	9	9
9	9	8	8	9	9	9	9

vely sort quadrants,
bottom ←:

2	3	2	2	2	3
3	3	4	5	5	6
4	5	6	6	7	7
8	8	9	9	9	9
0	0	2	2	1	0
3	2	5	4	4	3
5	5	9	8	7	7
8	8	9	9	9	9

Sort each column
in parallel:

1	1	0	0	2	2	1	0
1	1	2	2	2	2	2	3
3	3	3	3	4	4	4	3
3	4	3	3	5	5	5	6
4	4	4	5	6	6	7	7
5	6	5	5	9	8	7	7
7	8	8	8	9	9	9	9
9	9	8	8	9	9	9	9

Sort eac
alternate

0	0	0
3	2	2
3	3	3
6	5	5
4	4	4
9	8	7
7	8	8
9	9	9

uadrants,

:

2	2	3
5	5	6
7	7	7
9	9	9
2	1	0
4	4	3
7	7	7
9	9	9

Sort each column
in parallel:

1	1	0	0	2	2	1	0
1	1	2	2	2	2	2	3
3	3	3	3	4	4	4	3
3	4	3	3	5	5	5	6
4	4	4	5	6	6	7	7
5	6	5	5	9	8	7	7
7	8	8	8	9	9	9	9
9	9	8	8	9	9	9	9

Sort each row in p
alternately $\leftarrow$, $\rightarrow$:

0	0	0	1	1	1
3	2	2	2	2	2
3	3	3	3	3	4
6	5	5	5	4	3
4	4	4	5	6	6
9	8	7	7	6	5
7	8	8	8	9	9
9	9	9	9	9	9

Sort each column
in parallel:

1	1	0	0	2	2	1	0
1	1	2	2	2	2	2	3
3	3	3	3	4	4	4	3
3	4	3	3	5	5	5	6
4	4	4	5	6	6	7	7
5	6	5	5	9	8	7	7
7	8	8	8	9	9	9	9
9	9	8	8	9	9	9	9

Sort each row in parallel,
alternately $\leftarrow$, $\rightarrow$:

0	0	0	1	1	1	2	2
3	2	2	2	2	2	1	1
3	3	3	3	3	4	4	4
6	5	5	5	4	3	3	3
4	4	4	5	6	6	7	7
9	8	7	7	6	5	5	5
7	8	8	8	9	9	9	9
9	9	9	9	9	9	8	8

Sort each column
in parallel:

1	1	0	0	2	2	1	0
1	1	2	2	2	2	2	3
3	3	3	3	4	4	4	3
3	4	3	3	5	5	5	6
4	4	4	5	6	6	7	7
5	6	5	5	9	8	7	7
7	8	8	8	9	9	9	9
9	9	8	8	9	9	9	9

Sort each row in parallel,
alternately $\leftarrow$, $\rightarrow$:

0	0	0	1	1	1	2	2
3	2	2	2	2	2	1	1
3	3	3	3	3	4	4	4
6	5	5	5	4	3	3	3
4	4	4	5	6	6	7	7
9	8	7	7	6	5	5	5
7	8	8	8	9	9	9	9
9	9	9	9	9	9	8	8

h column

el:

0	0	2	2	1	0
2	2	2	2	2	3
3	3	4	4	4	3
3	3	5	5	5	6
4	5	6	6	7	7
5	5	9	8	7	7
8	8	9	9	9	9
8	8	9	9	9	9

Sort each row in parallel,

alternately $\leftarrow$, $\rightarrow$:

0	0	0	1	1	1	2	2
3	2	2	2	2	2	1	1
3	3	3	3	3	4	4	4
6	5	5	5	4	3	3	3
4	4	4	5	6	6	7	7
9	8	7	7	6	5	5	5
7	8	8	8	9	9	9	9
9	9	9	9	9	9	8	8

Sort eac

in paralle

0	0	0
3	2	2
3	3	3
4	4	4
6	5	5
7	8	7
9	8	8
9	9	9

2	1	0
2	2	3
4	4	3
5	5	6
6	7	7
8	7	7
9	9	9
9	9	9

Sort each row in parallel, alternately $\leftarrow$, $\rightarrow$:

0	0	0	1	1	1	2	2
3	2	2	2	2	2	1	1
3	3	3	3	3	4	4	4
6	5	5	5	4	3	3	3
4	4	4	5	6	6	7	7
9	8	7	7	6	5	5	5
7	8	8	8	9	9	9	9
9	9	9	9	9	9	8	8

Sort each column in parallel:

0	0	0	1	1	1
3	2	2	2	2	2
3	3	3	3	3	3
4	4	4	5	4	4
6	5	5	5	6	5
7	8	7	7	6	6
9	8	8	8	9	9
9	9	9	9	9	9

Sort each row in parallel,
alternately $\leftarrow$, $\rightarrow$:

0	0	0	1	1	1	2	2
3	2	2	2	2	2	1	1
3	3	3	3	3	4	4	4
6	5	5	5	4	3	3	3
4	4	4	5	6	6	7	7
9	8	7	7	6	5	5	5
7	8	8	8	9	9	9	9
9	9	9	9	9	9	8	8

Sort each column
in parallel:

0	0	0	1	1	1	1	1
3	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3
4	4	4	5	4	4	4	4
6	5	5	5	6	5	5	5
7	8	7	7	6	6	7	7
9	8	8	8	9	9	8	8
9	9	9	9	9	9	9	9

Sort each row in parallel,
alternately $\leftarrow$, $\rightarrow$:

0	0	0	1	1	1	2	2
3	2	2	2	2	2	1	1
3	3	3	3	3	4	4	4
6	5	5	5	4	3	3	3
4	4	4	5	6	6	7	7
9	8	7	7	6	5	5	5
7	8	8	8	9	9	9	9
9	9	9	9	9	9	8	8

Sort each column
in parallel:

0	0	0	1	1	1	1	1
3	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3
4	4	4	5	4	4	4	4
6	5	5	5	6	5	5	5
7	8	7	7	6	6	7	7
9	8	8	8	9	9	8	8
9	9	9	9	9	9	9	9

h row in parallel,

ely $\leftarrow$, $\rightarrow$:

0	1	1	1	2	2
2	2	2	2	1	1
3	3	3	4	4	4
5	5	4	3	3	3
4	5	6	6	7	7
7	7	6	5	5	5
8	8	9	9	9	9
9	9	9	9	8	8

Sort each column

in parallel:

0	0	0	1	1	1	1	1
3	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3
4	4	4	5	4	4	4	4
6	5	5	5	6	5	5	5
7	8	7	7	6	6	7	7
9	8	8	8	9	9	8	8
9	9	9	9	9	9	9	9

Sort eac

$\leftarrow$ or $\rightarrow$

0	0	0
2	2	2
3	3	3
4	4	4
5	5	5
6	6	7
8	8	8
9	9	9

parallel,

	2	2
2	1	1
4	4	4
3	3	3
6	7	7
5	5	5
0	9	9
0	8	8

Sort each column
in parallel:

0	0	0	1	1	1	1	1
3	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3
4	4	4	5	4	4	4	4
6	5	5	5	6	5	5	5
7	8	7	7	6	6	7	7
9	8	8	8	9	9	8	8
9	9	9	9	9	9	9	9

Sort each row in p
$\leftarrow$ or $\rightarrow$ as desire

0	0	0	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4
5	5	5	5	5	5
6	6	7	7	7	7
8	8	8	8	8	9
9	9	9	9	9	9

Sort each column
in parallel:

0	0	0	1	1	1	1	1
3	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3
4	4	4	5	4	4	4	4
6	5	5	5	6	5	5	5
7	8	7	7	6	6	7	7
9	8	8	8	9	9	8	8
9	9	9	9	9	9	9	9

Sort each row in parallel,
$\leftarrow$ or $\rightarrow$ as desired:

0	0	0	1	1	1	1	1
2	2	2	2	2	2	2	3
3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	5
5	5	5	5	5	5	6	6
6	6	7	7	7	7	7	8
8	8	8	8	8	9	9	9
9	9	9	9	9	9	9	9

Sort each column
in parallel:

0	0	0	1	1	1	1	1
3	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3
4	4	4	5	4	4	4	4
6	5	5	5	6	5	5	5
7	8	7	7	6	6	7	7
9	8	8	8	9	9	8	8
9	9	9	9	9	9	9	9

Sort each row in parallel,
$\leftarrow$ or $\rightarrow$ as desired:

0	0	0	1	1	1	1	1
2	2	2	2	2	2	2	3
3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	5
5	5	5	5	5	5	6	6
6	6	7	7	7	7	7	8
8	8	8	8	8	9	9	9
9	9	9	9	9	9	9	9

h column

el:

0	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	5	4	4	4	4
5	5	6	5	5	5
7	7	6	6	7	7
8	8	9	9	8	8
9	9	9	9	9	9

Sort each row in parallel,

$\leftarrow$ or $\rightarrow$ as desired:

0	0	0	1	1	1	1	1
2	2	2	2	2	2	2	3
3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	5
5	5	5	5	5	5	6	6
6	6	7	7	7	7	7	8
8	8	8	8	8	9	9	9
9	9	9	9	9	9	9	9

Chips ar

towards

parallelis

GPUs: p

Old Xeo

New Xe

	1	1
2	2	2
3	3	3
4	4	4
5	5	5
6	7	7
8	8	8
9	9	9

Sort each row in parallel,

$\leftarrow$ or $\rightarrow$ as desired:

0	0	0	1	1	1	1	1
2	2	2	2	2	2	2	3
3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	5
5	5	5	5	5	5	6	6
6	6	7	7	7	7	7	8
8	8	8	8	8	9	9	9
9	9	9	9	9	9	9	9

Chips are in fact e
towards having thi
parallelism and co

GPUs: parallel +
Old Xeon Phi: par
New Xeon Phi: pa

Sort each row in parallel,
$\leftarrow$ or $\rightarrow$ as desired:

0	0	0	1	1	1	1	1
2	2	2	2	2	2	2	3
3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	5
5	5	5	5	5	5	6	6
6	6	7	7	7	7	7	8
8	8	8	8	8	9	9	9
9	9	9	9	9	9	9	9

Chips are in fact evolving
towards having this much
parallelism and communicati

GPUs: parallel + global RA
Old Xeon Phi: parallel + rin
New Xeon Phi: parallel + m

Sort each row in parallel,

$\leftarrow$ or $\rightarrow$ as desired:

0	0	0	1	1	1	1	1
2	2	2	2	2	2	2	3
3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	5
5	5	5	5	5	5	6	6
6	6	7	7	7	7	7	8
8	8	8	8	8	9	9	9
9	9	9	9	9	9	9	9

Chips are in fact evolving towards having this much parallelism and communication.

GPUs: parallel + global RAM.

Old Xeon Phi: parallel + ring.

New Xeon Phi: parallel + mesh.

Sort each row in parallel,

$\leftarrow$ or $\rightarrow$ as desired:

0	0	0	1	1	1	1	1
2	2	2	2	2	2	2	3
3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	5
5	5	5	5	5	5	6	6
6	6	7	7	7	7	7	8
8	8	8	8	8	9	9	9
9	9	9	9	9	9	9	9

Chips are in fact evolving
towards having this much
parallelism and communication.

GPUs: parallel + global RAM.
Old Xeon Phi: parallel + ring.
New Xeon Phi: parallel + mesh.

Algorithm designers
don't even get the right exponent
without taking this into account.

Sort each row in parallel,

$\leftarrow$ or $\rightarrow$ as desired:

0	0	0	1	1	1	1	1
2	2	2	2	2	2	2	3
3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	5
5	5	5	5	5	5	6	6
6	6	7	7	7	7	7	8
8	8	8	8	8	9	9	9
9	9	9	9	9	9	9	9

Chips are in fact evolving
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Algorithm designers
don't even get the right exponent
without taking this into account.

Shock waves from subroutines
into high-level algorithm design.